1979

A study of the demand for money in Uruguay, 1957-1976

Alejandro Ramos
Iowa State University

A study of the demand for money in Uruguay,

1957-1976

by

Alejandro Ramos

A Thesis Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

MASTER OF SCIENCE

Major: Economics

Iowa State University
Ames, Iowa

1979

TABLE OF CONTENTS

Page

GLOSSARY

CPI	Consumer Price Index
DGEyC	Direccion General de Estadistica y Censos
IMF	International Monetary Fund
IFS	International Financial Statistics
GDP	Gross Domestic Product
GNP	Gross National Product
SAS	Statistical Analysis System

CHAPTER I. INTRODUCTION

An Overview of the Economic Process of Uruguay in the Period 1957-1976

The economic situation of Uruguay in the recent twenty year period can best be described as a combination of inflation and stagnation.

After the Second World War, an intense process of industrialization of the import substitution variety took place (much the same as in other Latin-American countries). It lasted until the beginning of the 1950s. After the Korean War (which provided a short export bonanza), alarming signs of growth retardation with inflationary pressures developed.

There were a number of reasons for this situation, of which the following are perhaps the most important:

The industrial sector had been subjected to indiscriminate subsidies and protected with high import barriers. This produced industries which faced little competition and had little incentive for efficiency and technological change.

The agricultural sector had seen growth halted long before. It produced beef and wool which, sold in the international market, provided most of the import needs of the industrial sector. It was based on extensive land use. This latter fact, along with the tax structure and perhaps risk considerations, also limited technological changes in this sector.

The government sector was then under pressure to provide jobs. Government intervention in economic activities was intense (electricity, telephone, waterworks, railroads, oil refining and retailing of oil products, insurance, commercial banking, etc., were operated by government corporations). Budgets inflated to compensate for losses by government corporations, partially explained by increased payrolls but also by the casual attitude of their managements regarding the need for efficient operation.

The inefficient behavior in both private and government sectors finally combined with the limit reached by the import substitution process.

Table 1 shows the evolution of real GDP. Total growth between 1957-1976 was 20.2 percent, which amounts to less than 1 percent per year. Even considering the low population growth, GDP per capita rose only 8.5 percent over the entire period, less than 0.4 percent per year. Growth in the industrial sector was the main factor explaining these figures. It went from an 8.8 percent annual average during 1945-1953 to 1 percent afterwards.

The consequences were important for employment, which had grown 45 percent during 1945-1955. As mentioned previously, the government sector partially absorbed the

Table 1. Uruguay: real GDP, population, real per capita GDP[a,b]

Year	GDP	Population	GDP/Population
1957	7,064.1	2,491	2.83
1958	6,809.3	2,517	2.70
1959	6,619.1	2,541	2.60
1960	6,859.2	2,567	2.67
1961	7,064.1	2,593	2.72
1962	6,908.5	2,619	2.64
1963	6,940.0	2,693	2.67
1964	7,057.1	2,619	2.69
1965	7,127.8	2,645	2.69
1966	7,389.0	2,671	2.77
1967	7,103.3	2,698	2.63
1968	7,184.9	2,725	2.64
1969	7,634.0	2,739	2.78
1970	8,001.4	2,752	2.91
1971	7,917.7	2,765	2.86
1972	7,634.0	2,765	2.76
1973	7,715.6	2,765	2.79
1974	7,960.6	2,765	2.88
1975	8,368.8	2,765	3.02
1976	8,491.3	2,765	3.07

[a]GDP in millions of N$ of 1975. Population calculated with data from 1963 and 1975 Census and assumptions regarding rate of growth (see Appendix B).

[b]Source: International Financial Statistics (17).

surplus labor during the following years.

Government deficits increased rapidly in the 1950s, resulting in rapid money stock expansion. This was probably at the root of the inflationary process which started in these years.

The average rate of inflation had been 5.5 percent during the decade 1940-1950, but early in 1951 a level of 21 percent was reached. By 1957, the rate was 13.9 percent and it never fell below double digits in the following twenty year period. The average annual rate for 1957-1976 was 48 percent, although 125 percent was reached in 1967-1968 and in the last five years of this period it averaged 76 percent (see Table 2).

A wage and price freeze was decreed in 1968, with the effect of lowering the rate of inflation to an average of 20 percent in the three following years. But as Table 3 shows, the slower pace was short lived, since increased government deficits brought a return to the previous high levels.

The financial sector of Uruguay

The financial sector consists of a commercial banking system (including an official bank, the Banco de la Republica), other institutions of the saving and loan type (the official Banco Hipotecario and the Caja Nacional de

Table 2. Uruguay: evolution of consumer[a] and wholesale[b] price indexes

Year	Wholesale price index	Consumer price index	Average annual rate of inflation[c] (%)
1957	-	.082	13.9
1958	-	.097	18.29
1959	-	.135	39.17
1960	-	.188	39.26
1961	-	.231	22.87
1962	-	.257	11.25
1963	.284	.311	21.01
1964	.422	.443	42.44
1965	.667	.693	56.43
1966	1.287	1.202	73.45
1967	2.178	2.274	89.18
1968	5.036	5.125	125.37
1969	5.806	6.200	20.97
1970	6.607	7.211	16.30
1971	7.956	8.938	23.95
1972	15.107	15.775	76.49
1973	32.469	31.075	96.98
1974	58.019	55.061	77.18
1975	100.00	100.00	81.61
1976	150.592	150.458	50.46

[a]Source: United Nations Statistical Bulletin (26).

[b]Source: International Financial Statistics (16).

[c]Defined as the variation in the consumer price index.

Ahorro Postal), the insurance bank (Banco de Seguros) and other small insurance companies, and the Stock Exchange.

The commercial banking system is of the branch type, and it had grown rapidly in the previous years. The halt in economic growth motivated that banks search for other types of activities to maintain profits.

Inflation, along with ceilings on interest rates produced negative real interest rates. To escape controls, a "parallel" banking system came into existence, formed by financial companies sometimes loosely tied to some banks. These companies were not closely supervised by the monetary authorities and dealt freely in the money and foreign exchange market.

These activities produced instability among financial institutions and in 1965 there were several bank failures. Banks started a process of mergers and branch closings after this year, seeking to improve the efficiency of their operations and reduce the needs for risky enterprises. The process of reorganization was slow, and there was another crisis in 1968-69.

The situation of the other financial institutions (Banco Hipotecario and Caja de Ahorro Postal) was also deeply affected by the inflationary process of the period. They had been very important in previous years in financing construction, but their long term policies in lending combined with the decrease in deposits deeply affected them. Assets decreased steadily so that new operations had to be cut. The situation was partially reversed after 1969 with the use of an indexed bond and indexed loans.

The Stock Exchange saw limited activities on private securities and bonds. Speculation in foreign exchange, real estate, or simply chanelling funds out of the country were superior financial investments. Towards the end of the period, activities centered around the new indexed bonds and a foreign currency government bond.

During the 1957-1976 period, the monetary authorities had two main instruments to control private credit expansion: the rate of discount and quantitative ceilings. On the other hand, it had no way to control government credit, and the Central Government borrowed heavily to finance deficits.

Authorities never used a money supply control type of policy, although credits to the private sector were tightened after visits of several IMF missions in the early 1960s.

The exchange rate regime was of the fixed type until 1972, and this system also produced swings in the money supply through the domestic effects of the balance of payments surpluses and deficits.

Devaluations were used on several occasions. They were necessary to alleviate the pressures created by the inflationary process. The most important devaluations were in 1959, 1965, 1967, 1968 and 1972. They temporarily relieved the economy by promoting exports, but import price raises caused new inflation and demand for higher wages, making the inflationary process suspect of self-generating by 1967-1968. The wage and price freeze was decreed in June 1968.

The period after 1968 was characterized by controls on wages and prices, use of a crawling peg exchange rate regime, and other forms of indexing in the money and capital markets.

In the real sector, efforts were directed to switch resources to the export sector, creating new market alternatives for the industrial sector. The response of the latter was slow mainly because of the dependence on foreign oil and technology as well as a slow rate of capital investment. The domestic capital market could not provide the funds for the necessary expansion rapidly, and foreign investment was very small.

In the domestic market, conditions didn't improve. A regressive redistribution of income characterized the period since 1957, but policy on wages accelerated these tendencies after 1972. This is shown in Table 3. Real wages fell by more than 40 percent in the twenty year

Table 3. Uruguay: real wage index (Base, 1957)[a]

Year	Real wage index
1962	86.5
1963	82.7
1964	80.1
1965	74.6
1966	73.8
1967	76.5
1968	67.9
1969	75.7
1970	74.70
1971	78.53
1972	65.10
1973	64.02
1974	63.47
1975	57.86
1976	54.48

[a]Source: Direccion General de Estadistica y Censos (6).

period, 31 percent between 1971-1976.

Firms which produced mainly for the domestic market were affected by this situation and could not always redirect resources to other activities. The consequences on unemployment were to keep it at high levels throughout 1972-1976.

Government deficits continued important after 1971 (see Table 4). Although lower real wages deflated the government payroll and the number of public employees tended to decrease, other reasons for increased expenditures existed. Among them were export subsidies and interest payments on foreign currency loans. Also tax revenues had been modified by lowering export taxes and creating the value added tax, but the results were slow and revenue didn't increase in real terms.

All of the above seeks to provide a framework of reference for this paper. The objective is to empirically estimate the demand for money in Uruguay in the period 1957-1976. The purpose is to take a closer look at the monetarist explanation of the inflation process, in which the demand for money plays a central role. If the demand for money proves to be stable as a function of a limited number of variables, and if supply of money can be controlled, then the equilibrium of the monetary sector can be predicted.

Table 4. Uruguay: government finance[a] (millions of N$ of 1975)

Year	Revenue	Expenditures	Deficit (-) or surplus	Deficit / Expenditure (%)
1965	909.1	1,183.3	-274.2	23.17
1966	1,123.1	1,148.1	-25.0	2.18
1967	733.6	1,090.6	-207.0	18.98
1968	950.2	1,030.2	-80.0	7.76
1969	996.8	1,187.1	-190.3	16.03
1970	1,149.6	1,264.7	-115.1	9.10
1971	1,153.5	1,625.5	-472.0	29.03
1972	1,062.4	1,236.7	-174.4	14.10
1973	1,191.3	1,308.1	-116.8	8.92
1974	1,067.7	1,429.1	-361.4	25.28
1975	995.2	1,333.6	-338.4	25.37
1976	1,144.3	1,352.9	-208.6	15.42

[a]Source: International Financial Statistics (17).

This provides an instrument for policy. The control of the money supply becomes an important instrument for reducing inflation.

If, on the other hand, the demand for money is not stable, the use of controls on the money supply is apparently less important for the control of inflation. Authorities would then have to search for some other instrument capable of obtaining the desired objectives.

In the following Chapter II a demand for money function is presented and controversial issues regarding its formulation discussed. Money is defined for practical purposes, and also different approaches are presented. Chapter III presents the model utilized in this study to obtain an empirical demand for money function for Uruguay during the period 1957-1976, also some previous research results on Latin America demand for money functions are examined. Chapter IV involves the analysis of the results of the empirical work done, and Chapter V presents a set of limitations of the analysis.

CHAPTER II. THEORETICAL ASPECTS OF THE MODEL

There are a number of questions which arise when demand for money functions are formulated. In the past, the main questions have been related to their stability and what variables should be included in the function. In the more recent past, while these problems remain current, other issues were raised: the proper functional form of the function, the correct use of distributed lags to estimate it, the use of a demand-supply adjustment mechanism, etc.

The reasons for this discussion (together with extensive empirical research) lie in the central point that monetarist thinking assigns to the demand for money. If the set of properties theoretically desirable are found in practice, and if money supply can be controlled by the monetary authorities, then a causal relationship can be established between the quantity of money in existence and the level of prices and nominal income. Friedman (11, 12) and others have attempted to provide such a framework, putting the amount of money in the economy and its growth rate as the main instruments to achieve stabilization.

Another body of thought (which will be loosely called "Neo-Keynesians") formulates a different demand for money function. Money is considered just one of several financial assets available in the economy, and its demand more closely related to the expenditures of the economic unit. The stability of the demand for money is not such a crucial relationship for this body of thought, and in relative terms other relationships could prove more reliable for policy.

This is, of course, one of the points of dispute between these two groups in the recent past, but one that has caused considerable controversy.

In the following, the main formulation of the demand for money function is presented according to monetarist lines. Later, Neo-Keynesian critiques will be presented, together with the main features of their contrasting view.

In the monetarist line, stability of the demand for money function is an empirical property of crucial importance. Stability is not taken to mean absolute constancy of course, but rather constancy in the relationship. But as Friedman (12) remarks, there is no easy way out by stating the need for a stable relationship and then declaring an indefinitely large number of variables. For the stability concept not to be emptied of empirical content, the number of independent variables has to be kept small. Friedman states that the following variables should be included in a complete demand for money equation: (a) the nominal return on bonds, (b) the nominal return on equities, (c) the expected rate of change in the price level, (d) the ratio of human to nonhuman wealth, (e) expected income, and

(f) tastes and preferences of the public.

Money is regarded as a form of storing wealth, but there are other forms of holding wealth available to the economic unit. Bonds, securities, and durable goods might constitute alternative forms. Therefore, it is through the study of the changes in the forms of holding wealth that the demand for money can be identified. These changes in turn depend on the optimizing behavior of the economic units. Given the risk and liquidity preference of the units, the behavior will be to equate the marginal return obtained in the holding of each form of wealth. One basic element in the difference among the components of wealth is the different form of the income stream associated with each type. Whether it is a bond or an equity, the income stream will be composed of fixed payments in nominal units or in shares of the returns of enterprises. Each form of wealth has a certain "utility" of which the income stream is an integral part.

This optimization process is subjected to the constraints imposed on the substitution of one form of wealth for another.

The inclusion of the expected nominal returns on bonds and equities occupy the place of the substitutes in the general theory of demand of any good. The rate of change in a price level is a similar concept, since it seeks to reflect the change in the return of the holding of physical goods in general. The services of these goods are in kind, rather than in money, and could be considered to be a percentage of its original purchase value.

However, this "real" yield is only a portion of the total yield applicable, since variations in prices determine "adjustments" in the value of the services of physical goods. In addition, physical goods, like equities, are regarded to yield a nominal return due to appreciation or depreciation in nominal value.

Expected variations in the nominal return of money are also included by Friedman in the function. This return could be zero or a positive or negative amount, and will depend on the definition of money used and what assets are included as "money".

Friedman says that institutional constraints limit the conversion of human into nonhuman wealth. Since the human wealth involves earning capacity as the major asset, changes in the proportion of such types of wealth in total wealth would involve variations between the composition of assets, therefore should be included in the demand for money function.

Total wealth is also an argument, setting a constraint to the demand for money as in the general theory of demand. It is the total that has to be divided into the different kinds of assets. Since in practice it is difficult to estimate it, it is approximated by permanent income.

Permanent income is preferred by Friedman following his studies on the consumption function. It seeks to eliminate the fluctuating components of current income, therefore provides a longer term concept. Friedman's demand for money function can then be represented as:

$$\frac{M}{P} = f(y, w, r_m, r_b, r_e, \frac{1}{P}\frac{dP}{dt}, U)$$

where:

M/P = real cash balances

y = permanent income

w = ratio of human to nonhuman wealth

r_m = expected nominal return on money

r_b = expected nominal return on bonds

r_e = expected nominal return on equities

$\frac{1}{P}\frac{dP}{dt}$ = expected rate of change of prices of goods

U = other variables that affect utility of money and are not related to income (tastes, preferences, etc.)

The expression shows two interest rates, r_b and r_e for specific types of assets, but there is one general rate applicable to all assets implicit in the calculation of permanent income (y). Friedman states that this general rate is a kind of weighted average of the two previous rates plus the rates applicable to human wealth and physical goods. Since the latter two cannot be observed directly, it could be assumed that they vary with r_b and r_e. Then, a simplification of the function at this stage would mean that only two nominal interest rates are considered, r_b and r_e.

Friedman shows that in a world with no differences of opinion about price movements and interest rate movements, and bond and equities were equivalent except that the former are expressed in nominal units, then arbitrage would make the streams of return on each form of holding wealth equal.

In addition, he shows that if the rates of interest are stable or changing at the same percentage rate then the nominal interest on bonds equals the nominal interest on equities plus the rate of inflation. Therefore, under these ideal conditions, one interest rate variable would be enough to represent the return on wealth in the demand for money function.

The above interpretation has followed Friedman's exposition very closely. Its final product is a functional relationship that links the real amount of cash demanded to a measure of wealth (approximated by permanent income) and a measure of the opportunity cost of holding money. This opportunity cost is represented by a single interest rate in the final equation, but this carries forward to the empirical problem of which one to use.

In this interpretation there is no distinction among the different reasons for holding money. Every unit of money is regarded as performing a variety of services, and the holder of money equates at the margin the value of each unit of money to each unit of other forms of wealth.

This interpretation relies heavily on Walras' Law as it is commonly formulated. In equilibrium, the flow excess demand for all goods and services in the economy is zero. In other words, every participant in the exchange process obtains an equal value for the goods he supplies. Money is typically thought of as a residual of the exchange process, the "residual" good. Every participant would be left with certain bundles of goods and services and a certain amount of money. This amount of money constitutes his stock demand for money.

When Friedman equates the services of money to those of goods in general, he has a Walrasian framework of reference in which the excess demand for money equals the excess supply of all the rest of the goods lumped together. In this context, an increase in the demand for money can only be made possible by decreasing the demand for all other goods together. This of course can be achieved by decreasing the demand for the i^{th} good holding the demand for all the other (n-1) the same.

This view is also called the "asset" demand for money, underlying the fact that money is generally held for a variety of purposes and not for certain specific functions and that, in this respect, it resembles all other goods.

Given that a stable demand for money function could be obtained, and a few independent variables isolated, other problems still stand in the way.

There exists a problem of aggregation. The demand for money of a single unit when aggregated over all units in the economy could introduce the aggregation problem. The same amount of money could be distributed in a variety of ways among all the units. If it is assumed that only the total amount matters, the aggregation problem plays no role. This is the path followed by Friedman, who prefers not to attack the problem directly.

Also, given the properties of the demand function, and that it is an unobservable entity, the supply of money has to meet certain requirements. Empirical work is capable of estimating the equilibrium of the money market. To be able to identify the demand function for money, the supply function has to be affected by some variables other than those which affect the demand for money. If the supply is on the contrary affected by the same variables, demand cannot be identified.

The usefulness of the concept of the demand for money also lies at the heart of this problem. What matters is observing the effects of changes in the supply of money on the equilibrium levels in the money market. This process is what hopefully sheds some light on the mechanism of variation of prices and nominal income.

There exists another step in the process of using the money supply, even when all the above theoretical considerations are met. Monetary authorities have to be able to effectively control it. This is also a crucial issue, and it has received considerable attention.

Neo-Keynesian thinking present a different theoretical framework for the analysis of the demand for money. It distinguishes the different motives for holding money, of which the "transactions" motive is the most important. Keynes had assigned great importance to the "speculative" motive, but modern thinking downgrades it because of the existence of riskless financial assets which dominate money at any level of risk consideration.

Basically, the transactions demand for cash balances states that money is demanded for purposes of spending it. The demand for money is then more an "ex-ante" concept. The holder of money is anticipating to spend a certain amount in goods and services and "demands" money according to this level of anticipated expenditures. The existence of "asset" aspects of the demand for money are not neglected in theory, but its importance will vary given the institutional conditions prevailing (existence of riskless bonds, related to the "speculative" purpose) and the risk preference of the holder (precautionary purpose).

For this body of thought, what would set money apart from other goods is its unique property as a medium of exchange. When monetarists consider "assets" purposes only and refer to the process of holding money as an "ex-post" concept, the transaction purpose for holding is virtually eliminated.

This is a basically different approach, even when monetarists state that all motives for holding money are given equal importance. This is not the case with the transaction motive. The use of a residual demand for money prevents the inclusion of a transactions motive, because the transactions among the different economic units have already taken place and will not be resumed until the next "auction", following the traditional Walrasian example.

For Neo-Keynesian thinking, the demand for money is rather an "ex-ante" concept. Money is held primarily to be used in spending, so it is directly linked to the volume of planned expenditures in an immediate future.

Money shares with the rest of the goods some basic properties, but its importance as a medium of exchange and unit of account are stressed. The uniqueness of money lies in the fact that it provides the main services to the holder when leaving his possession, and not while remaining in his command.

The above interpretation would directly reject the application of Walras' Law to the demand for money, because money is not demanded because other goods are supplied, but because other goods are also demanded. Only by maintaining a residual type of demand for money can be Walras' Law sustained in this case. With a transactions approach it can't be used because it would violate the conditions for which money is demanded.

Even when accepting the existence of other than transaction motives for holding money, other critiques could be made against the use of Walras' Law in the formulation of the demand for money. In a Walrasian world, transactions are settled simultaneously at the same set of prices. But in reality this is not so. The sum of the values of the goods and services taken from the market could be different from what it is supplied to the market.

In other words, even the "residual" demand for money is not necessarily linked to the excess supply of all other goods in a unique way. Variations will depend on the extension of the income period, the number of transactions on the same goods carried forward by the individual, and the difference in the prices of the same goods transacted in the income period.

In the same line of thought is the critique motivated by the existence of unemployment, -- that is, when disequilibrium is present. The existence of involuntary unemployment (excess supply of labor) would imply in a Walrasian world that there exists an excess demand for goods and services (including money) on the part of the unemployed. But in the real world, this implicit excess demand for goods and money can't become effective unless the unemployed sell their services first.

In another line of reasoning, it can be stated that Neo-Keynesians do not trust the stability of the demand for money. Keynes had originally stated the theoretical possibility of the "liquidity trap", mechanism through which the demand for money became infinitely elastic at a positive interest rate. This possibility is currently rejected, but the existence of major displacements in the demand for money function are accepted. Certain variables can play an unexpected role in the function and later drop out of the picture. The search for another instrument in relation to output and employment policies provides the consumption function as a more stable relationship, and fiscal policy is favored.

Some Neo-Keynesians have found that the demand for money is empirically affected by other variables than those stated by Friedman. Tsiang (26) in particular found that the volume of foreign trade relative to GNP was relevant in a study for Taiwan. The inclusion of such a variable was an attempt to account for the rapid growth of the foreign sector in that country, within a transactions approach to holding money. This is so because the volume of transactions is normally related to GNP in a proportional way. But when the number of foreign transactions increase and GNP stays at a lower level in relative terms, then GNP is not a good proxy for total transactions in the economy. Tsiang's results were affirmative of the importance of such a variable and he concluded that a demand for money function defined with only a measure of income was potentially unstable. Exports are not a stable function of income.

The argument that this could be a rather unique example is not appealing. Many countries are involved at present in export promotion policies that could eventually produce the same phenomenon. In many developed countries, extensive empirical work establishes sound relationships between the stock of money on one side, and a measure of wealth and the opportunity cost on the other. However, recent experience in the United States prove that very sound and stable relationships of the above kind can change rather abruptly. Goldfeld

(14) shows that the demand for money in this country has shifted as a result of the incidence of still largely unknown forces.

The above suggests that the stability of the demand for money is not a settled matter. Empirically, the observance of a stable relationship for some sample period does not rule out instability with respect to the same variables in a subsequent period. In theory it is always possible to include a larger number of variables, but Friedman already warned against that.

In consequence, all results of the empirical work need to be relativized.

Another final interesting point should be stressed. Even when the above discussion suggests a major gap between the two most widely accepted interpretation, the actual form of the function in the empirical verification does not differ much. Both approaches utilize an interest rate variable as a measure of opportunity costs, and a measure of income is also included. The only difference of importance is the use of a permanent income concept on the monetarist side, because the idea is to obtain a proxy for wealth, the real constraint on the demand for money. On the Neo-Keynesian side, the income variable is current income as a proxy for the level of transactions. But even when this could provide a way of discriminating (given the different formulations of current and permanent income), a closer look shows that, since permanent income is approximated with the use of weighted past values of current income, and that Muth (20) showed that expected income provides an unbiased forecast of current income, both concepts are not so far apart.

There is also another difference in the way money is defined, since monetarists prefer in general the inclusion of time deposits. The following section deals with this and related problems.

A Definition of Money for Practical Purposes

In the above presentation of the discussion, one important point has been omitted. Money has not been properly defined.

What is going to be understood for money in this work is crucial to the conclusions, since stable relationships with the independent variables could possibly not exist when some financial assets are included in the definition.

The problem of the definition of money has stirred a great deal of controversy among economists in the past. Rapid changes in the institutions of the financial system and the changing importance of some financial assets within the system are the possible explanations.

During the last century, the discussion focused on the growing importance of deposit liabilities of the banking system. It appeared convenient to broaden the definition of money to include them.

In the recent past, the growth of financial intermediaries has been the problem, since their liabilities sometimes possess properties that constitute a close alternative to those of money narrowly defined. The best known example in the United States is the growth of the saving and loan associations.

The problem has received theoretical attention but there is no definitive conclusion as to definition of money is the most appropriate. The reason for this is partially related to the above discussion of the stability of the demand for money. It was said that the property of stability was a necessary condition for monetary policy, but that stability in a very large number of variables would complicate it or make it uninteresting. A definition of money that includes too many close substitutes constitutes a problem for policy. There might be too many instruments with possible desirable effects in theory, but in practice the partial effects could be limited. On the other hand, if financial assets with a very close substitutability are not included, then the relationship could be unstable.

A trade-off then exists in the search for a stable demand for money function between a sufficient number of close substitutes in the definition and the need to keep them small in number.

Four main schools of thought have developed concerning this issue, as classified by Harry Johnson. The discussion is taken from Feige (9).

One, which is the most commonly used, is to include in the definition only currency outside banks and demand deposits. The basic argument for this approach is that those are the two assets which function directly as means of payment. It has an advantage in the sense of avoiding the complexities of all the wide range of assets that don't constitute final means of payment. It is the most simple and pragmatic approach.

Friedman and Meiselman reject this position and consider that there is no need to have an a priori defensible criterion. They argue in favor of an empirical decision concerning the assets that should be included in the definition of money. In their work, after some experimentation with different financial assets, they decided to include commercial banks time and savings deposits in the definition, together with currency outside banks and demand deposits. They found that variations in time deposits were better correlated with variations in income than variations of other financial assets. Also, the close substitutability of time deposits with the other components already in the definition was an important reason.

Feige criticizes this approach. He suggests that in demand for money studies, the use of an interest rate variable to measure the opportunity cost of money could have ambiguous effect when time deposits are included in the definition of money. The rate of interest would also be a proxy variable for the yield on time deposits.

The third position is represented by John Gurley. He also rejects the need for an a priori theoretical defensible definition. On the contrary, he suggests that any financial assets could be defined as money. Every financial asset should have a weight attached according to its degree of substitutability with money and demand deposits.

The fourth approach is represented by the Radcliff Report. "Liquidity" is regarded as the relevant monetary magnitude but they never offered an explitic operational definition of the term.

Feige observes that the explicit recognition of the substitutability of the liabilities of the financial intermediaries respect to currency and demand deposits is not a sufficient argument to include them in a broader definition of money. An alternative would be to let their rates of return be arguments in a regression were the dependent variable is some quantity easily controlled by the monetary authorities.

Feige then suggests the most sensible solution is to define money narrowly and then to take explicit account of all the financial assets that could affect money so defined. Every component of money should be a dependent variable. Currency, demand deposits and time deposits are quantities that may be affected by policy, so they should be the dependent variables. When the inclusion of some substitutes in the regression fails to produce significant results, they could be dropped from the relationship.

The interest of this last approach is that it keeps the definition simple and yet includes the complexities of the structure of the financial markets in developed countries.

Feige's approach could be thought unimportant in the case of underdeveloped countries with less complex financial markets. However, there always seem to be close substitutes playing a role.

In Uruguay, a bond issued by the mortgage bank (Banco Hipotecario) serves as an example. This bond was indexed in 1969, and this represented an attractive feature given the prevailing high rates of inflation and the existence of ceilings on nominal interest rates on time deposits. The circulation of this bond increased quickly so that it became an important element of the public's portfolios. According to the above analysis by Feige, if time deposits are included in a definition of money, then the circulation of this bond should be used as one of the independent variables, in the demand for money, given its close substituting properties.[1]

Other financial assets that become close substitutes for money in countries with high rates of inflation are foreign currency and foreign currency deposits. If the exchange rate is fixed, the need for periodic devaluations brings quick profits to the foreign currency holder. So, there is a periodic flight from money and return later after the devaluation. This process took place in Uruguay during part of the sample period, then it would be of interest to include an estimate of the foreign currency in existence at every given moment. However, this information is practically impossible to obtain.

The above suggests that even with a smaller choice of assets in the financial market there is still a problem in selecting the best definition of money. Even Feige's criteria is not applicable due to the nonmeasurability of some assets like the circulation of foreign currency.

The problem is worse in countries which are not fully monetized, in which extensive use of wages in kind and the existence of an important subsistance sector are found. In this case, the problem with the definition of money would be

---

[1]The mortgage bank operates as a broker, so the public regards the holding of a bond as the holding of a deposit. It is more liquid than a time deposit in the sense that it can be sold at any time, but the brokerage fee puts a constraint to its liquidity.

that even the use of currency and demand deposits is not very extensive. This is not the case of Uruguay.

The definition of money used in this study is the first of those discussed above. Money will represent the sum of currency outside banks and demand deposits. It provides a good starting point because, following Feige's suggestion, the degree of accuracy of a regression using this definition could be eventually improved with the inclusion of other financial assets as independent variables.

In other words, even when in this work the financial assets included will be limited to currency and demand deposits, there is built in flexibility to include more assets later as independent variables. In the mean time, those two variables constitute a good framework for policy actions if the demand for money so defined is a stable relationship and the other conditions are also met.

CHAPTER III. THE SPECIFICATION OF THE DEMAND FOR MONEY MODELS

The present study will present a model for the demand for money in Uruguay for the period 1957-1976. The purpose is to explore the stability condition through the recurrence to a key concept: the expected rate of inflation.

Uruguay presented an annual average rate of inflation of 48 percent during the 1957-1976 period. This important rate suggests that it is the main component in the measure of the variations in the opportunity cost of holding money. In the previous discussion, Friedman stated that a final global measure with the use of just one interest rate would be the choice. The rate of inflation is a proxy for the opportunity cost of holding money in this study. This is important because in Uruguay like in other Latin-American countries, nominal interest rates are bounded by the monetary authorities.

The recurrence to the rate of inflation as a measure of the opportunity cost of holding money draws from the work of Cagan (1) on hyperinflations, and it has also been used in other works in Latin-America. Deaver (4), Diz (7) and Corbo-Lioi (3) use it in their demand for money functions for Chile and Argentina.

Real income grew less than 1 percent annually in Uruguay in the sample period. The rate of inflation also overwhelms the changes in this variable, but it seems important to provide an estimate of the income elasticity of the demand for money. Cagan didn't use an income variable in his work, but the short period of the hyperinflations advised so. The other authors mentioned always include an income variable.

If the income variable is understood as a proxy for transactions, then current income should be included. If, on the other hand, a proxy for wealth is needed, then permanent income would be the choice. Both alternatives will be followed in this work.

The elimination of the use of a nominal interest rate in the formulation of the demand for money function produces two different problems. The first is that all the financial assets not considered in the definition are lumped together with the rest of the physical goods in the economy in the same group. In other words, only two categories of assets are considered; money and all the rest of the goods.

The second problem is that using the rate of inflation as a proxy for the opportunity cost of holding money implies that this cost would be zero in the absence of inflation. This is not true, and emphasizes that the approach is valid only for high rates of inflation. If inflation drops to zero, there is simply no measure provided.

Also, a crucial qualification has to be made at this stage. It is not the actual rate of inflation in a given period what matters to the money holder, but what he expects it to be in a given future period. What the rate of inflation really was will be discovered later. Future rates are forecast by the holder of money according to how the information available to him affects his expectation.

In several models it has been traditional to use an adaptive type of process in the formation of the public's expectation. The idea is that the public adjusts expectations slowly in response to changes in reality. A number of previous observations in the variable have a weight assigned, and every new observation carries important observation but changes the new forecast by a fraction of the previously expected values. In other words, the structure of the weights is such that economic actors adapt slowly to changes; the units never learn on committing mistakes; the process of forecasting new values produces an error every time.

This process is represented by a geometrically decaying weight structure on the past observed values of the variable, in the present case the rate of inflation. Applications are found in consumption and investment models. In monetary models the classic example is Cagan's work on hyperinflations. The above adaptive expectations scheme is represented as:

$$i^*_t - i^*_{t-1} = d(i_t - i^*_{t-1})$$

where

i^*_t = expected rate of inflation at time t

i_t = observed rate of inflation at time t

d = parameter representing the coefficient of expectation, normally a value between 0 and 1.

The expected value of the rate of inflation at time t is what it was expected in the previous period plus a percentage of the difference of what the actual rate is and what was expected in the previous period.

Values of d close to 1 represent a very short period in the formation of expectations, values close to 0 the opposite.

The use of this mechanism is of course one of the possible ways to model the public's expectations. There is a degree of arbitrariness in the scheme, the limitations introduced will be discussed later in this work. A number of authors have used it in Latin American studies and they provide a point of reference for this work.

The adaptive expectation mechanism is not restricted to the rate of inflation variable, it is also used to approximate the permanent income variable. This follows from Friedman's consumption function work and serves as a proxy for wealth, as discussed in the previous chapter.

The Model

The model used in this study can be stated as:

$$M_t^d = \alpha + \beta i_t^e + \gamma Y_t^e \qquad (1)$$

where

M_t^d = logarithm of real per capita demand for money

i_t^e = expected rate of inflation

$= \log\ (p_t^e/p_{t-1}^e)$

p_t^e = expected level of prices at t

Y_t^e = logarithm of real per capita expected GDP

β = expected cost elasticity

γ = permanent income elasticity

All the variables are represented in natural logarithms, and this permits the interpretation of the coefficients of i_t^e and Y_t^e as elasticities.

Since expected variables are not observable, the previously discussed adaptive mechanism has to be put into the picture to arrive at a form usable in regression.

The use of this mechanism will introduce lagged variables in the equation as will be shown later, but still there exists another nonobservable variable in the above formulation, the demand for money.

The model then is taken to represent the equilibrium of the monetary sector by introducing the supply of money.

In the past, some studies did not show this fact explicitly and the use of distributed lags of the money stock variable in the regressions produced confusion in the application of the adaptive mechanism scheme.

The reason for this situation is also related to the way the supply of money enters in the equation. It can be assumed to be instantaneously equal to demand, or the existence of a demand-supply partial adjustment mechanism can be utilized, in which case the demand adjusts to a fraction of the changes in supply in a given period.

The correct form of implementation of the demand-supply partial adjustment mechanism is to use an equation like this:

$$M_t^d = M_{t-1}^d + k(M_t^s - M_{t-1}^d) \qquad (2a)$$

where

M_t^s = log of real per capita supply of money at time t

k = demand supply partial adjustment coefficient

The value of the parameter k will normally be expected to be between zero and one. Values close to one reflect a very short lag in the response of demand to supply variations, values close to zero the opposite.

In the literature, this mechanism was frequently implicitly expressed in a different way:

$$M_t^s = M_{t-1}^s + k'(M_t^d - M_{t-1}^s) \tag{2b}$$

In this formulation, used by Feige (8), Chow (2) and others, the supply of money appears receiving the effects of changes in demand and partially adjusting to its variations in a given period.

This last form is incorrect, as Starleaf (24) observed. The money supply is the true independent variable in the formulation of the model, so the specification of a mechanism like (2b) does not truly specify a demand-supply partial adjustment mechanism since the effects are in fact reversed. The use of (2b) determines that k' be an over-adjustment demand supply coefficient as Starleaf shows.

Equation (2) above then represents the true formulation of the mechanism. It can also be expressed like this:

$$M_t^d = k(M_t^s + (1-k)M_{t-1}^s + (1-k)^2 M_{t-2}^s + \ldots + (1-k)^r M_{t-r}^s) \tag{2c}$$

This alternative formulation also shows that demand can be represented as a geometrically weighed average of present and past values of the supply of money. In other words, the same pattern of weight distribution is used here and in the implementation of the adaptive expectations scheme discussed before. In this present example, the consequence of the application of a demand-supply partial mechanism to the structure of the model will be the presence of distributed lags in the dependent and independent variables in the reduced form of the model.

This discussion helps to clarify the methodological problem that arises when studies of the demand for money function depart from a reduced form equation which included distributed lags of the dependent variables. Feige (8) shows that a specification problem would exist in those studies, because the use of distributed lags could be rationalized with reference to either a demand-supply partial adjustment mechanism or adaptive expectations in the implementation of one or more independent variables.

To avoid this specification problem, the correct approach he suggests is to introduce the structural relationship and then substitute in it the different assumed behavior of the expected variables and the demand response to supply. This will be the method followed in this work.

A specification of the supply of money function is needed at this stage. The earlier discussion made clear the need for a supply of money as function of some other variables than the demand. Most studies consider an exogenous form for the specification of the supply of money, and this is the approach that will be followed here. The limitations of this procedure will be discussed later. It could be stated at this stage that by taking an exogenous value for the money supply, the particular extreme assumption that there are no variables that jointly affect the demand and supply is used.

$$m_t^s = \bar{m}_t \qquad (3a)$$

where:

m_t^s = nominal supply of money at t

Then

$$M_t^s = \log \frac{m_t^s}{p_t}$$

In order to complete the model, it is necessary to include the explicit formulation of the adaptive expectations scheme in the expected rate of inflation and permanent income variables. This is accomplished by means of the following two equations:

$$i_t^e = \lambda(i_t + (1-\lambda)i_{t-1} + (1-\lambda)^2 i_{t-2} + \ldots)$$

$$= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j i_{t-j} \qquad (4)$$

$$Y_t^e = \delta(Y_t + (1-\delta)Y_{t-1} + (1-\delta)^2 Y_{t-2} + \ldots)$$

$$= \delta \sum_{j=0}^{\infty} (1-\delta)^j Y_{t-j} \qquad (5)$$

This formulation can be represented the way it was done before, following the original work of Koyck:

$$i_t^e = i_{t-1}^e + \lambda(i_t - i_{t-1}^e) \qquad (6)$$

$$Y_t^e = Y_{t-1}^e + \delta(Y_t - Y_{t-1}^e) \tag{7}$$

The model formed by Equations (1) through (5) is the most general which can be obtained with the above assumptions. It will provide, after all the substitutions are completed, a general reduced form equation which with appropriate statistical tools will produce estimates of all the structural parameters. In the following, all the required substitutions to arrive at the reduced form are carried forward.

We first obtain from Equation (2a) above:

$$M_t^s = \frac{1}{k} M_t^d - \frac{(1-k)}{k} M_{t-1}^d \tag{8}$$

Iterating backwards Equation (1), both expressions for M_t^d and M_{t-1}^d can be substituted into (8) to get:

$$M_t^s = \frac{1}{k}(\alpha + \beta i_t^e + \gamma Y_t^e) - \frac{(1-k)}{k}(\alpha + \beta i_{t-1}^e + \gamma Y_{t-1}^e) \tag{9}$$

Reorganizing Equation (9):

$$M_t^s = \alpha + \frac{\beta}{k} i_t^e + \frac{\gamma}{k} Y_t^e + (1 - \frac{1}{k})\beta i_{t-1}^e + (1 - \frac{1}{k})\gamma Y_{t-1}^e \tag{10}$$

Now that the partial adjustment mechanism has been included, the expected variables have to be substituted. Using the expressions (4) and (5) lagged one period when applicable:

$$M_t^s = \alpha + \frac{\beta\gamma}{k}(i_t + (1-\lambda)i_{t-1} + \ldots) + \frac{\delta\gamma}{k}(Y_t + (1-\delta)$$

$$\cdot\ Y_{t-1} + \ldots) + (1 - \frac{1}{k})\beta\lambda(i_{t-1} + (1-\lambda)i_{t-2} + \ldots)$$

$$+ (1 - \frac{1}{k})\gamma\delta(Y_{t-1} + (1-\delta)Y_{t-2} + \ldots) \qquad (11)$$

If (11) is iterated backwards one period and the result multiplied by $(1-\lambda)$ and then subtracted from (11), we get:

$$M_t^s - (1-\lambda)M_{t-1}^s = \alpha\lambda + \frac{\beta}{k}\, i_t + (1- \frac{1}{k})\beta\lambda i_{t-1} + \frac{\gamma\delta}{k}(Y_t$$

$$+ (1-\delta)Y_{t-1} + \ldots) + (1 - \frac{1}{k})\gamma\delta - (1-\lambda)\frac{\gamma\delta}{k})$$

$$\cdot\ (Y_{t-1} + (1-\delta)Y_{t-2} + \ldots) - (1-\lambda)(1 - \frac{1}{k})\gamma\delta$$

$$\cdot\ (Y_{t-2} + (1-\delta)Y_{t-3} + \ldots) \qquad (12)$$

This equation has to be iterated backwards once, multiplied by $(1-\delta)$ and the result subtracted from (12) to obtain:

$$M_t^s - (1-\lambda)M_{t-1}^s - (1-\delta)(M_{t-1}^s - (1-\lambda)M_{t-2}^s)$$

$$= \alpha\lambda\delta + \frac{\beta\lambda}{k}\, i_t + (1 - \frac{1}{k})\beta\lambda - (1-\delta)\frac{\beta\lambda}{k})i_{t-1} - (1-\delta)$$

$$\cdot\ (1 - \frac{1}{k})\beta\lambda i_{t-2} + \frac{\gamma\delta}{k}\, Y_t + \frac{\gamma\delta}{k}\,(-(1-\lambda) + (k-1)Y_{t-1}$$

$$- \frac{\gamma\delta}{k}(1-\lambda)(k-1)Y_{t-2} + u_t \qquad (13)$$

Equation (13) can be used in regression. It is assumed not to hold exactly and a random term u_t added. It contains eight variables and therefore it will provide nine reduced form estimates of the parameters. Since there are only six structural parameters the equation is overspecified. The attempt to use standard linear regression theory would yield contradictory estimates of the structural parameters. Consequently, nonlinear methods of estimation have to be used (see Appendix A on Statistical Methods).

The addition of a stochastic term to Equation (13) introduces the problem of the assumption regarding its behavior. Appendix A also presents a discussion of the most crucial assumption in these type of models, the serial independence of the components of the error term. There it is shown that the presence of autocorrelated disturbances in a model with lagged values of the dependent variable will fail to produce estimators with the desired statistical properties. Therefore, some degree of correction for autocorrelation has to be allowed in the model.

Following the original recommendation of Fuller and Martin (13), a first order serial correlation scheme is assumed to govern the formation of u_t:

$$u_t = \rho u_{t-1} + e_t \qquad (14)$$

where

ρ = serial correlation coefficient

e_t = random error with mean zero, finite variance and free of autocorrelation.

To obtain the final full reduced form of the model, there is to iterate (13) backwards once, multiply the resulting expression by ρ and then subtract the result from (13). These calculations provide:

$$\begin{aligned} M_t^S = {} & \alpha\lambda\delta(1-\rho) + \frac{\beta\lambda}{k} i_t + \frac{\beta\lambda}{k}(k-2+\delta-\rho) i_{t-1} + (-(1-\delta) \\ & \cdot \frac{\beta\lambda}{k}(k-1) - \rho((k-1) - (1-\delta))) i_{t-2} + (1-\delta)(k-1) \\ & \cdot \frac{\beta\rho\lambda}{k} i_{t-3} + \frac{\gamma\delta}{k} Y_t + \frac{\gamma\delta}{k}(k-2-\rho+\lambda) Y_{t-1} \\ & + (-(1-\lambda)(k-1)-\rho((k-1)-(1-\lambda)))\frac{\gamma\delta}{k} Y_{t-2} \\ & + ((1-\lambda)(k-1))\frac{\gamma\delta\rho}{k} Y_{t-3} + (2+\rho-\lambda-\delta) M_{t-1}^S \\ & + (-(1-\gamma)(1-\delta) - ((1-\lambda) + (1-\delta))) M_{t-2}^S \\ & + ((1-\lambda)(1-\delta))\rho M_{t-3}^S + e_t \end{aligned} \qquad (15)$$

This equation is also nonlinear in the structural parameters but there are more reduced form parameters than before since the number of variables is now eleven.

Other assumptions could be made regarding the behavior of u_t. In particular, higher order degrees of autocorrelation could be assumed. This however adds considerable computational expense because, as it is explained in Appendix A, the numerical optimization methods that have to be used are expensive.

The model in this general form has the advantage of including all the elements common to many demand for money models used by others in the recent past. It has the necessary flexibility to test different hypotheses regarding the structural parameters, some of them following the previous theoretical discussion, and some following other researchers experiences in the past.

One of the possible forms of introducing these different hypotheses is by means of constructing submodels which include one or more fixed values of some of the structural parameters. This methodology will also allow comparisons with previous demand for money studies in other Latin American countries. By proceeding from the most complex structure to the more familiar reduced forms used in the past the risks of using simpler formulations can be evaluated.

In effect, many researchers in the past have been chosen to depart from a reduced form equation in which some distributed lagged variables were included. The estimates of the parameters which result are not readily identifiable in terms of a given structure, and this is an important limitation of econometric work.

The present methodology implies the different submodels will be plausible if the assumed simplifying hypotheses are confirmed statistically. For example, the model that assumes no partial adjustment mechanism will be valid if the full model regression casts doubt regarding its usefulness.

Submodel 1. Instantaneous equilibrium between supply and demand for money:

The mechanism of partial adjustment might not be operative, which would be statistically represented by a value of the parameter k very close to one. This was the experience of Starleaf (24) in his study of U.S. data between 1952-1966. In Latin-American studies, only Corbo-Lioi (3) includes it in the model. All the rest of the studies mentioned above provide for instantaneous equilibrium between supply and demand for money. In other words, there is no lag in the response of demand to supply.

The necessary change in the model involves replacing Equation (2) in the model with:

$$M_t^s = M_t^d \tag{16}$$

But the same result in the reduced form equation can be obtained by observing that (2) and (16) coincide for k=1. The reduced form equation for this submodel is then:

$$M_t^s = \alpha\lambda\delta(1-\rho) + \beta\lambda i_t + \lambda\beta(\delta-1-\rho)i_{t-1} + \lambda\beta(\rho(1-\delta)$$

$$\cdot\ i_{t-2} + \gamma\delta Y_t + \gamma\delta(-1-\rho+\lambda)Y_{t-1} + \rho(1-\lambda)\gamma\delta Y_{t-2}$$

$$+ (2-\rho-\lambda-\delta)M_{t-1}^s + (-(1-\lambda)(1-\delta) - \rho((1-\lambda)$$

$$+ (1-\delta)))M_{t-2}^s + ((1-\lambda)(1-\delta))\rho M_{t-3}^s + e_t \qquad (17)$$

Equation (17) has a nonlinear form also.

Submodel 2. Partial adjustment using current instead of permanent income:

Most of the discussion regarding the use of permanent or current income in the demand for money function applies to the controversy on the concept of money as an asset. When it is regarded as the analogue of a consumer durable good, Friedman's consumption theory recommends the use of permanent income. If money is viewed primarily as an asset for "transactions" purposes, then current income is a better proxy for transactions than permanent income.

Apparently, the inclusion of current income could bring additional light to the discussion. However, expected income can be regarded as a proxy for permanent income and also as a forecast of current income under certain conditions expressed by Muth (20). He proves that if the change in current measured income is a first order moving average of random errors, then the adaptive expectations mechanism used to approximate permanent income provides an optimal forecast of measured income.

At any rate, there is considerable interest in providing for a "current" income hypotheses because the adaptive expectations mechanism could possibly not be a good description of the process which determines permanent income. For example Vogel (29) in his cross section of 16 Latin American countries found no evidence for supporting the permanent income hypotheses in a regression with the rate of inflation as dependent variable (see below for a discussion of Vogel's result).

The reduced form equation for this submodel is obtained by taking $\delta=1$ in the reduced form (15) of the full model:

$$M_t^S = \alpha\lambda(1-\rho) + \frac{\beta\lambda}{k} i_t + \frac{\lambda\beta}{k}(k-1-\rho) i_{t-1} + \frac{\beta\lambda}{k}(-\rho(1-1))$$

$$\cdot\ i_{t-2} + \frac{\gamma}{k} Y_t + \frac{\gamma}{k}(k-2-\rho+\lambda) Y_{t-1} + (-(1-\lambda)(k-1)$$

$$- \rho(k-2+\lambda))\frac{\gamma}{k} Y_{t-2} + ((1-\lambda)(k-1))\frac{\gamma\rho}{k} Y_{t-3}$$

$$+ (1+\rho-\lambda) M_{t-1}^S + (-\rho(1-\lambda) M_{t-2}^S + e_t \qquad (18)$$

This model presents nine variables and five structural parameters, so it is also overidentified.

Submodel 3. Instantaneous equilibrium of supply and demand, current instead of permanent income:

This submodel corresponds to the long run explanation of the money market as stated in Equation (17), but this time in terms of current income. The formulation of the reduced form version of the sub-model is easily obtained setting k=1 in Equation (18), alternatively $\delta=1$ in (17) or both in (15). The result is as follows:

$$M_t^S = \alpha\lambda(1-\rho) + \beta\lambda i_t - \beta\lambda\rho i_{t-1} + \gamma Y_t + \gamma(1-\rho-\lambda)Y_{t-1}$$

$$+ (1-\lambda)\gamma Y_{t-2} + (1-\rho-\lambda)M_{t-1}^S - \rho(1-\lambda)M_{t-2}^S + e_t \qquad (19)$$

This model is also overspecified, presenting five parameters and seven variables.

Submodel 4. Partial adjustment in the money market but no income variable in the equation:

It is considered important to present a submodel without the inclusion of an income variable. It would provide a measurement of the explanatory power of the expected rate of inflation alone, then closely resembling Cagan's model on hyperinflations. Cagan (1) did not include income in his demand for money function because of the short period of time in which the hyperinflation processes occurred. He also justified his decision by pointing at the highly unequal magnitude of the variation in the rate of inflation compared with any possible change in real income.

In the present study the period analyzed is much longer and the rates of inflation observed never reached the proportions of those observed by Cagan.[1] However, the real increase in GDP of the Uruguayan economy has been very slow in the sample period as was discussed in Chapter I. Therefore, even when the observed rates of inflation have been much lower than in a hyperinflation process, the same argument regarding the disproportion between the increases in the rate of inflation and income growth might be used. The sole use of the expected rate of inflation is then justified.

The elimination of the income variable also has some practical advantages. Some data problems are solved, since there is no quarterly GDP information and the assumption made regarding the quarterly distribution of its variation is only a proxy.

Also, using a transactions version of the demand for money, the use of current income would normally serve as a

---

[1]Cagan defines hyperinflation when the rate of inflation reaches the level of 50 percent monthly.

proxy for the level of transactions in the economy. However, the important variation in real wages in the period which amounts to serious changes in the distribution of income would probably make necessary the use of other variables to represent transactions. This point will be discussed further later in the study.

The reduced form equation for this model is by making $\gamma=0$, $\delta=1$ in the full model (15), obtaining the following expression:

$$M_t^S = \alpha\lambda(1-\rho) + \frac{\beta\lambda}{k}\, i_t + \frac{\beta\lambda}{k}(k-1-\rho)\, i_{t-1} + \frac{\beta\lambda}{k}(-\rho(k-1))$$

$$\cdot\ i_{t-2} + (1-\lambda+\rho)M_{t-1}^S - \rho(1-\ )M_{t-2}^S + e_t \qquad (20a)$$

Submodel 5. <u>Instantaneous equilibrium in the money market and no income variable in the equation</u>:

This model is basically the same as the previous one except that the partial adjustment mechanism is assumed not to be operative.

There is another reason to present it, it provides the first identified model of those so far discussed. Appendix A presents another form of dealing with serial correlation which is only applicable to linear models. The model resulting after the parameter ρ is also eliminated, will have three structural parameters remaining in the equation which will produce three reduced form estimates, therefore there will not be contradictory estimates of the structural when a regular linear regression is used.

The reduced form is obtained by making $\gamma=0$, $\delta=1$, $\rho=0$ and $k=1$ in the general reduced form (15), to get:

$$M_t^S = \alpha\lambda+\beta\lambda i_t + (1-\lambda)\overline{M}_{t-1} + e_t \qquad (20b)$$

Although the structural parameters can be disentangled, the model is still basically nonlinear and the variances of the structural parameters are not trivial expressions of the variances of the reduced form parameters. Numerical approximation methods would have to be used, but the easiest way is to use a Taylor series expansion of the nonlinear expression.

All the results of the empirical work performed on these submodels and the full model are presented in the following chapter.

An Overview of the Main Models Formulated in the Past with Latin American Data

It is convenient to contrast the models presented above with previous work in countries with conditions similar to those of Uruguay. A revision of the work for other countries like the U.S. would be too lengthy and cumbersome. Besides, the studies that will be mentioned follow the same body of theory. The only important difference in theoretical terms is the more frequent use of variables like the rate of inflation in the regression instead of interest rate variables.

Most of these studies were done in the 1960s, in the middle of the controversy between monetarists and structuralists on the causes of inflation. Typically regressions are performed on reduced form models. The rate of inflation or the money stock is the dependent variable. The independent variables are one of these two, according to which one is used as the dependent variable and also real income, minimum wages, variation in the foreign exchange rate and lagged values. The objective was to obtain stable functions in formulations that included or excluded structuralist variables according to the inclination of the author.

The choice of the dependent variable was not considered a problem. It was just a matter of selection of one or the three alternatives: changes in the rate of inflation, real cash balances or income velocity. The three forms were similar in the sense that the supply of money is the real independent variable in the model. So it was just a matter of what particular variable in the model was more important for the purpose at hand.

However, problems arise when dynamic considerations enter the picture. Later it will be discussed how a body of modern theory rejects the alternative use of the rate of inflation and the money stock as dependent variables in a model explaining the monetary sector. There is a causation problem involved, so that each form could be more correct depending on the possible feedback of the rate of inflation on the money stock.

Harberger's study (16) is the first of importance (in an empirical sense) which was done in Latin America. He used the rate of inflation as the dependent variable. The independent variables are the money stock, real income and the "acceleration" variable, which is the change in the rate of inflation. Also, a minimum wage variable is included as a way of testing a structuralist argument. Lagged values of the money stock are also included on the right hand side of the equation. This was rationalized by means of a previous exploratory regression of the rate of inflation on the money stock. Lags higher than two did not prove significant, so only two lags were included in the main regression. His results were consistent with the monetarist interpretation of inflation, since the wage variable was not found to be significantly different from zero.

The use of distributed lags in the money stock in Harberger's model was an ad-hoc procedure. The mechanism of adaptive expectations was mentioned but not explicitly introduced in the model. There is no systematic lag between supply and demand allowed. These two mechanisms could be operative in which case the use of distributed lags in a reduced form model presents a case of the well-known identification problem discussed earlier.

Also, Harberger's model implicitly includes distributed lags in the dependent variable which has more serious effects on the results. The "acceleration" variable is eliminated early in his study because of lack of significance, but its presence in a regression equation estimated by ordinary least squares produces inconsistent estimates even in large samples and also invalid t and F tests. The latter applies for the case in which serial correlation is present in the model, which is the standard situation when lagged variables are used in time series models.

When the acceleration variable is eliminated in the model, the estimates will be consistent but the statistical inefficiency will still be present.

All the above implies that the nonsignificance of some variables in this context can be misleading. There are too many statistical problems involved to produce a fair degree of confidence in the results.

Diz (7) and Diaz-Alejandro (5) present similar models for Argentina in different periods. Diaz-Alejandro uses other independent variables like the variation in the exchange rate and in the hourly nominal wage rate in industry, and he finds them statistically significant. The use of an income distribution variable had to be rejected, and he found that lagged variables presented either small values or had the wrong sign. Even when his results supported structuralist positions, his empirical work suffers from the same defect than Harberger's. His results with the use of lagged variables suggest a nonlinear model in certain unknown structural parameters.

Diz used constrained nonlinear least squares approach and obtained satisfactory results within a monetarist framework. Deaver (4) work on Chilean data is similar. Both refer the use of lagged variables to an adaptive expectations mechanism which is explicitly included in their work. However, the presence of serial correlation in this framework in which lagged dependent variables are also included result in inconsistency and inefficiency as in the models discussed earlier. This fact affects the validity of their results also.

Vogel (29) presented one of the most recent studies (1974) of the topic, a pooled study for sixteen Latin American countries. The rate of inflation was the dependent variable. There were few observations because the data were annual, but the reliance on a pooled study presumably could compensate for that fact.

The model was essentially Harberger's without the minimum wage and change in the rate of inflation variables. Therefore all the comments made to Harberger's model are also applicable.

Vogel found the coefficient for lagged current income to be negative in his regression. He found this fact particularly hard to explain since it would reject the permanent income hypotheses in the demand for money. He then proceeded to explain permanent income in a different way than the traditional distributed lag of values of current income, stating that in Latin America a better proxy is the change in income.

This result does not follow from his empirical work. Equation (15) in this study (see above) presented the full reduced form of the model. It can be seen that with the methodology used, there are a variety of values of the structural parameters for which the resulting values of the reduced form parameters of lagged income could be negative. But this fact does not imply that the coefficient of expected income be equal to any given value, or that the distributed lag approach to permanent income is incorrect.

The problem is of course again the use of reduced form equations in which the reduced estimated parameters are assumed to produce significant information regarding the lagged variable to which they are connected. But this is a very risky approach and very unreliable in general terms.

Another recent work of the Chilean experience is by Corbo-Lioi (3). It is a part of a broader econometric model for Chile. The money stock was the dependent variable and an adaptive expectation mechanism was used along with systematic lags in demand, so his model was similar to the one used in this study. Some of the empirical findings also share the same type of problems, the estimation of the income elasticity of the demand for money was limited by the small variation in real income in the sample period. The problem of serial correlation is explicitly confronted and a similar solution given to it.

In conclusion, there is no evidence in the models presented so far that permit a definitive solution to the old monetarist-structuralist controversy. Even when several models were capable of isolating one or two key variables, like the rate of inflation when money stock was the dependent variable and vice versa, all the models in general present some statistical problems. Identification problems along with serial correlation in the presence of lagged dependent variables produce highly undesirable results to be able to discriminate among models, so the variables which are excluded from the final regressions because of apparent lack of significance could possibly not be so in a different context. Also, the high explanatory power of the regressions would be much decreased with correction for autocorrelation and the conclusions of the studies changed if the underlying structures are explicitly identified.

In this sense, it is evident that the empirical work on the demand for money or the causes of inflation is still in an early stage in Latin America, and will probably continue to be there until more serious statistical efforts are achieved.

CHAPTER IV. ANALYSIS OF THE RESULTS OF THE EMPIRICAL STUDY

The results of the regressions performed are summarized in Table 5 for the period 1957-1976. All the models were estimated using logarithmic transformations of the variables. Therefore, the coefficients β and γ are the opportunity cost elasticity and the real income elasticity of the demand for money, respectively.

The use of a logarithmic transformation of the data is common in studies on the demand for money. However, linear or log-linear forms of the demand for money are sometimes encountered. The matter of the appropriate functional form was not thought to be an important analytical problems until recently.

Zarembka (30) showed that the functional form chosen could have an important bearing on the selection of variables in the regression. A discussion of his conclusion will be presented in the next chapter.

Before presenting the results it is important to remember that a global minimum of the least squares cannot be reached with total certainty in nonlinear models as it is in linear models. The results are presented under the assumption that in fact the global minimum has been reached (see Appendix A).

The results will be presented following the order of presentation of the model and submodels in the previous chapter.

Full Model

The most general model presented is represented by the reduced Equation (15) above. Its performance is one of the weakest in the empirical work done for this study. The model has an asymptotical F value nonrejectable at the 1 percent level, but when the performance of the individual variables is assessed the results are not encouraging. All the estimates except two are smaller than twice their respective standard errors, meaning that the t values are small and the variables are statistically nonsignificant (see Table 5).

This model and the first submodel below are the ones which presented more considerable estimation problems. The reason for this is connected to the high degree of multicolinearity (the matrix X'X is near singular in both models). This fact makes the matrix inversion process in the computer extremely slow and expensive, therefore it is advisable to leave the full model at this incomplete stage.

The idea is then to check the sign and dimensions of the coefficients in relation to what would normally be expected

Table 5. Analysis of results[a]

Model	Parameters							F value
	α	β	λ	γ	δ	k	ρ	
Full model	-0.84 (0.187)	-4.67 (2.95)	0.137 (0.10)	0.167 (1.10)	0.73 (1.63)	0.823 (0.15)	0.29 (1.7)	204.19
Submodel 1	-0.85 (0.13)	-4.16 (1.35)	0.195 (0.06)	-0.367 (1.01)	0.68 (2.65)	-	0.301 (2.65)	239.76
Submodel 2	-0.85 (0.14)	-4.31 (1.63)	0.156 (0.07)	-0.06 (0.83)	-	0.85 (0.14)	0.43 (0.14)	232.67
Submodel 3	-0.86 (0.11)	-3.98 (1.18)	0.20 (0.06)	-0.39 (0.77)	-	-	0.46 (0.12)	277.94
Submodel 4	-0.85 (0.14)	-4.39 (1.44)	0.15 (0.05)	-	-	0.85 (0.13)	0.43 (0.13)	283.82
Submodel 5 (nonlinear)	-0.85 (0.12)	-4.29 (1.19)	0.18 (0.05)	-	-	-	0.45 (0.12)	350.74
(linear)	-0.39 (0.49)	-3.68 (1.14)	0.23 (0.05)	-	-	-	-	155.44

[a]Values in parentheses are estimated standard errors of the coefficients.

and leave the burden of statistical significance to the rest of the submodels.

The estimate for the intercept is negative and small in absolute value.

The cost elasticity estimate is -4.67. It shows the correct sign although the absolute value is larger than expected. It is higher than both the values of -1.476 and -1.537 reported by Deaver (4) for Chile (1932-1957) and Diz (7) for Argentina (1938-1962). The reasons for this result are probably connected to the different model used in this study, and also to the higher average rate of inflation for Uruguay in the sample period.

The coefficient λ is the elasticity of the rate of inflation expectations and turns out to be 0.137 for the full model. This compares with 0.10 and 0.075 in Chile and Argentina in the above mentioned studies. This figure for Uruguay means that on the average it takes around 5 years to form rate of inflation expectations, or 20 quarters. This is longer than expected, since the high rates of inflation would presumably speed up the process and shorten the period.

When both the cost elasticity and elasticity of rate of inflation expectation are taken together, it shows that while the first is higher (in absolute value) than what was found by Deaver and Diz, the second is lower. If differences in the building of the models are set aside, the results point at a different form of response of the public in Uruguay, one for which the framework of reference for inflation changes slower but when finally some possible "threshold" points are reached, the demand for real cash balances react very strongly.

The elasticity of permanent income is positive but very low. This result is common in Latin American studies, and it could be related to the income distribution. If increases in real income are associated with a more unequal distribution of it, and if lower income groups in the economy have higher preference for holding real cash balances instead of other forms of wealth, then permanent income elasticities would be low. A transaction framework would produce the same result since a more unequal income distribution would mean fewer transactions in the economy (cash balances are economized).

The elasticity of income expectations is 0.73 for the full model. This result compares with 0.469 for the United States (1952-1966) as reported by Starleaf (24). The period of formation of expectations is less than a year.

The demand-supply adjustment elasticity is 0.82, this also represents a very short period of adjustment. This result is highly plausible since the high rates of inflation in the period made the costs of being out of equilibrium higher than normal. Corbo-Lioi (3) uses the same adjustment procedure and finds similar results, with an average lag of a year and a half. Starleaf's point estimate for the United States was found to be above 1, this shows that both Chile and Uruguay's demand for money function lag slightly behind the supplies but not significantly so.

The point estimate for the first order serial correlation coefficient is 0.29, meaning that it is sizable. However, this result as well as all the previously discussed are as it was mentioned just points of reference since they present little statistical significance.

Submodel 1. Instantaneous equilibrium between supply and demand for money:

This model also presented estimation problems as it was mentioned before.

All the results are similar to the full model, except for the negative income elasticity estimate. This result complements the above discussion.

Statistically this model performs a little better since all the standard errors are low except for the two troubled coefficients. However, the low speed of convergence made necessary to stop the iteration process earlier than in the rest of the submodels, so results have less validity (although they are not meaningless).

Submodel 2. Current instead of permanent income:

This model provides an estimate of the current income elasticity of the demand for money. It does much better in the statistical sense and at the same time produces estimates for the structural parameters which do not differ much from those of the full model and Submodel 1.

The intercept is again negative and significantly different from zero. The cost elasticity estimate is -4.31 but this time this value is less than twice its standard error.

The elasticity of rate of inflation expectations is slightly higher than before, pointing at a faster formation of expectations.

The only important difference is the negative estimate for the current income elasticity which is, however, not significantly different from zero. As it was mentioned in the Introduction of this study, real income changed very little in the sample period. This fact, together with the need for interpolation to obtain a quarterly series are the probable reasons for such a low value which fails to be significant.

Also, the explanation used before when discussing the permanent income elasticity also applies. It seems that the inclusion of an index of the variation in the income distribution could help in providing a better estimate of the income elasticities. But again, given the little variation in real income, presenting both variables the net effect could be small. An attempt will be made later to include an index for income distribution but without the inclusion of a real income variable at the same time.

The model in current income is satisfactory in statistical terms, and this is an important fact. The asymptotic value of F is significant at the 1 percent level, all the variables are significant except real income. It confirms the previous point estimate of the partial adjustment elasticity and also it fails to be significantly different from 1, which means that the hypothesis of instantaneous equilibrium cannot be rejected.

Since both the coefficients for the income elasticity and the partial adjustment elasticity fail to be significant, the model can be improved by getting rid of them in the final equation. This is done in terms of Submodels 3 and 4 below.

Submodel 3. Instantaneous equilibrium between supply and demand. Current instead of permanent income:

As Table 5 shows, the results do not change from what was seen above in an important way.

The cost elasticity is lower but still represents a high value. This is explainable because when demand and supply for cash balances are forced to be instantaneously equal the cost of being out of equilibrium is zero. This cost is a component of the inflation cost of holding real balances. This would also explain the fact that the cost was already low since the lag in demand to supply was very short.

The use of a demand supply partial adjustment mechanism also could answer a fraction of the difference found in the estimates of cost elasticities in this work and in the studies by Deaver and Diz. If they had used a mechanism like this, their estimates of the cost elasticity would probably be higher than what they reported.

Submodel 4. Partial adjustment in the money market with no income variable in the equation:

As it was said, there is need to eliminate the income variable for lack of significance. Other methods would have to be created to obtain a reliable estimate of the income elasticity, and the sample period would have to include more variation than the observed.

This methodological step is of considerable interest as it was expressed before because of the similarities with Cagan's model. The elimination of income from the equation leaves just one current variable, the rate of inflation, in the model.

However, the model is still nonlinear in the structural parameters. Cagan's model does not include an adjustment coefficient; the next submodel eliminates the parameter k by assuming instantaneous equilibrium and will therefore be closer to Cagan's.

There are two minor differences in the result from what was discussed before. The cost elasticity estimate is slightly higher than the one in the previous Submodel 3, and closer to the one in Submodel 2. This apparently validates the above interpretation that the cost of being out of equilibrium in the money market is playing a role in the cost elasticity.

The other minor difference is in the estimate of the elasticity of rate of inflation expectations: it is slightly lower than in the previous submodel and also closer to the one in Submodel 2. But the variation is not significant statistically.

The rest of the estimates present no change at all. The estimate for the first order serial correlation remains quite high at 0.43 like in the two previous submodels. This is an indication of the importance that the correction presents in the model.

Submodel 5. Instantaneous equilibrium in the money market and no income variable in the equation:

As outlined above (Chapter III), a version of this submodel can be estimated by use of linear regression theory. The step of elimination of the adjustment mechanism in the Submodel 4 above leaves an equation with 4 structural parameters and 5 reduced form estimates of them. This is over-identified also, therefore nonlinear methods have to be used to estimate it. However, there is an alternative method to correct autocorrelation which makes unnecessary the use of the autocorrelation scheme presented above. The coefficient ρ then is set equal to zero, and the new form of this submodel is then linear. This is so because the elimination of ρ does away with two reduced form estimates, the equation will have three structural parameters and three reduced form estimates of them. There will be a unique way to disentangle the structural coefficients and so the model is just identified. Once it is estimated, serial correlation can be corrected for with the method presented in Appendix A for linear models.

The two approaches to estimation of this submodel are presented in Table 5. The reason for this presentation is that it could provide an idea of the modifications that would appear in the previously discussed model if a more effective or general procedure to deal with serial correlation were available for nonlinear models. The method for linear models provides a rather inexpensive way to deal with serial correlation of various orders since this is the actual approach of the successive steps of correction.

The version of this submodel in which serial correlation is corrected by means of ρ in the equation (the nonlinear form) presents little change from the above. All the coefficients are significant and of the same absolute value. The estimate of ρ is high. These results are confronted with the third step of correction of the linear version of the same submodel. The final remaining serial correlation was left at 0.08, but of course the process of correction could continue.

It is interesting to observe that the estimates do not change in an important way. The cost elasticity is lower in absolute value, but not significantly so. The elasticity of expected rate of inflation expectations is higher, pointing at a faster process of formation of the expectations. This is important because both results are more plausible than before. The cost elasticity appeared too high and the formation of expectations too slow given the rates of inflation observed in the period. The reason for this is in part the existence of serial correlation, although the effects are not significant in the selection of the variables. All the coefficients remain higher in absolute value than twice their standard errors, except for the intercept term.[1]

This result is very appealing in the sense that it points at proportionality between the money stock and the expected rate of inflation. This result belongs to the monetarist tradition, but still the high value of the cost elasticity of the demand for money is not very encouraging for that interpretation. The reason for this cannot be shown in this paper and is just a matter of speculation. The use of another sample period with much less average rate of inflation could prove useful in determining the evolution of the cost elasticity of the demand for money. If it decreases in an important way (in absolute terms) then this would mean that it fluctuates widely, changing the position of the curve. In other words, it wouldn't be very stable.

---

[1]The perfectly identified version of the model can be estimated by means of a linear regression, but it remains nonlinear. The standard deviations cannot be disentangled in the same way from the estimated standard deviations of the reduced coefficients. A Taylor linear expansion was used and the resulting estimates of the standard errors of the structural parameters are then approximate.

This is what Neo-Keynesians fear and the reason why they prefer to rely on other relations.

However, it remains true that for high rates of inflation the predictive power of the rate of inflation is very high in the demand for money. This is probably the reason why monetarist thinking attributes total power to the money supply in fighting inflation in countries like Uruguay.

The lag in the formation of expectations would also be a signal that to have a reasonable success, policies of tight money have to be kept for more than four years. This will be discussed further later.

The equation presents a lower F value but still significant at the 1 percent level.

Model in Expected Inflation and an Income Distribution Index

The previous model attempted to provide an estimate of the income elasticity of the demand for money. The reasons for its apparent failure were connected to the particular conditions of real income growth in the sample period and in information problems. The problem of income distribution was also mentioned as one of the possible reasons for the low elasticities values found.

The present model seeks to provide a partial answer to the question of what are the effects on the demand for money of the changes in the distribution of income.

The index for income distribution was constructed using a quarterly real wage index over the real per capita income index. Therefore, the limitations in the construction of the real per capita income index will also be included here. The available information was for the period 1968-1976. This period is shorter than what was used in this study, therefore it could provide some insight on the changes, if any, of the last years.

The model with income distribution produces several changes in the results (see Table 6). The elasticity of real cash balances respect to income distribution is positive, although not significantly different from zero at the 5 percent level.

The positive sign obtained points at the right direction if a transaction approach is used in the analysis of real cash balances. The assumed behavior of the lower income groups in the economy is to hold more cash as a proportion of their income, because expenditures are a higher proportion of income than the groups with higher income. Then, a redistribution of income in their favor would imply larger cash balances at the same level of real income. The opposite would be true for a redistribution in the other direction, so the positive sign obtained is highly plausible.

Table 6. Analysis of the income distribution hypotheses in the model (1968-1976)

Model	Parameter						F value
	α	β	λ	γ	θ[a]	ρ	
Submodel 3 (1968-1976)	-0.72 (0.13)[b]	-3.40 (1.06)	0.26 (0.06)	-2.74 (0.93)		0.27 (0.22)	126
Model with income distribution index	1.67 (0.70)	-5.42 (2.40)	0.16 (0.06)	-	0.09 (0.14)	0.29 (0.20)	115

[a]θ = income distribution elasticity of the demand for money.

[b]Values in parentheses are standard errors of the coefficients.

Friedman expresses that changes in income distribution affect the demand for money function in his theoretical framework, but prefers to downgrade this effect, probably assuming that its effects would be small.

Other changes in the estimated coefficients are present in the intercept and in the cost elasticity estimate. Comparing this model with the equivalent Submodel 3 estimated for this subperiod (Submodel 3 presents current real income where the present model has the income distribution index), it is interesting to notice how the intercept goes from negative to positive, and significantly so at the 5 percent level.

The direct effect of inflation is higher as estimated by the cost elasticity, and the elasticity of rate of inflation expectations is smaller than when the income variable is present, but the differences are not significant at the 5 percent level.

A by-product of the above estimation of Submodel 3 for this comparison is the possibility of looking at the possible changes motivated by changes in the sample periods.

Comparing Submodel 3 for 1968-1976 with the previous estimation of its for the whole period produces little variation in the estimates. There is a slight increase in the coefficient of expectation that points at a shorter period of formation of expectations in the rate of inflation.

This is normal because of the higher average rate of inflation of the subperiod over the average for the whole 1957-1976 period.

The current income elasticity is higher in absolute value than before. This evidently is the consequence of the exclusion of changes in the income distribution. Some degree of inelasticity of the demand for real cash balances in respect to income is likely to be present though but it is not adequately measured here and in most of the Latin American studies. The measurement problems and the availability of the required data are also recurrent problems in those studies, so it seems premature to be able to verify the low income elasticity of the demand for money in Uruguay and other countries in the area.

<u>General remarks on the results obtained</u>

The models and submodels utilized perform reasonably well, but many of the important topics raised in the methodological section of this paper remain in the dark.

It is not possible to obtain a reliable estimate of the current and expected income elasticity and the overall importance of the income redistribution process that took place in the second half of the sample period cannot be fully analyzed.

The use of a real wage over GDP index as constructed is probably just a rough approximation to income distribution, since other groups with nonwage type of income would have to be included and not just lumped together with the other category.

Also, the use of both real income and an income distribution index together in a formulation of the demand for money function could probably shed more light on the effects of income distribution on the estimated elasticity of real income. The model so constructed would be more complicated if the methodology is maintained, otherwise it would require a different formulation.

The problem of the stability of the demand for money has to be related to the effects of the use of different sample periods on the estimated parameters. The results above show a partial acceptance of the stability hypothesis when the period 1957-1976 is used. But this result was obtained in the presence of high rates of inflation, and it remains to be seen what would be the behavior of the function with lower rates.

The short period 1968-1970 in which the effects of the wage and price freeze were felt do not provide enough degrees of freedom to test the model obtained in the full period. The model should be tested for previous years to the sample period, to include a large sample and contrast the results with periods in which the rate of inflation was lower. It seems likely that in those cases the importance of the expected rate of inflation would be lower, opening the door to the possibility of "threshold" effects. This type of effects represents still another hypothesis regarding the behavior of the public in the formation of expectations. Below a certain rate of inflation, the public may believe that either the level is not substantial or that it will decrease and so the decisions are not affected by it. So that real cash balances held by the public are weakly related to changes in the rate of inflation. But if inflation is persistent for a given time and the rates are higher or at least do not decrease, the public may suddenly become aware that it probably is to stay and then begin to actively include expectations of its future levels in their behavior toward money.

Of course, if the rate of inflation is low or near zero, other forms of measuring the opportunity cost of holding money become important, variations in the real return on capital would have to be faced since the inflation rate changes would not overwhelm its changes anymore.

The above discussion suggests that the form of the demand for money function assumed in this study would have no predictive power in the absence of a substantial rate of inflation, and that the utilization of a certain measure of the interest rate would have to be faced not to lose generality.

In the meantime, provided that the rates of inflation remain around the average of the sample period used, the predictive power of the expected rate of inflation covers a great portion of the variance in the real stock of money in the Uruguayan economy.

The public seems to form a process of expectation on the inflation rate in around 4 years, which marginally could explain why the significant decrease in the inflation rate during 1968-1970 did not modify the results obtained.

There seems to be room for other explanatory variables, but it is hard to evaluate them given the slow change in the economic variables in the sample period. It is evident that the rate of inflation is about the only variable that presents important fluctuations in the period, as it was explained in the Introduction.

Other variables that could prove important in the following years and the future are real income (which has grown faster in recent years), and a measure of the importance of international trade.

The results obtained by Tsiang (26) with this last index applied to the Taiwanese experience are extremely important. He found this variable significant in his regression study of the demand for money, and this points at a potential unstable element in the above formulation. Exports in particular do not depend on income, and the serious export promotion in which Uruguay and other countries in Latin America are involved could produce similar results in a few years.

If this result is proved true, it would represent a blow to the stability hypotheses but it would be closer to a transaction framework of the demand for money. In effect, the increase in trade relative to GDP means an increase in the volume of transactions and if this factor is not accounted for, then the use of real income would be a poor proxy for transactions.

If all of these elements are put in the picture, the traditional monetarist notion of a stable demand for money function would probably have to be redefined, but this is rather a speculative point at this stage, and future work has to be done to provide the answer.

CHAPTER V. LIMITATIONS OF THE ANALYSIS

The model utilized in this study presents a set of conditions and assumptions determining the validity of the results obtained. An explicit determination of the main factors that limit the analysis allows both a better appraisal of the results and at the same time directs attention to where the efforts of future empirical work should be concentrated.

Since some of the limitations do not involve theoretical questions, it is convenient to subdivide them in the following fashion: 1) limitations that affect the performance of the model without involving the methodology used; and, 2) methodological decisions that affect the selection of the model.

Limitations that Affect the Performance of the Model Without Involving the Methodology Used

In this group one of the easiest recognizable elements is:

Data and measurement problems

Availability of data was not a problem except for a quarterly series of GDP, which effected the estimation of the income elasticity. The information regarding quarterly real wage for the period before 1968 was not received in time for this study.

Measurement problems seem to be more important. The transactions version of the demand for money relies on a proxy variable for its determination, and the variable normally chosen is income. However if the income distribution is changing rapidly in an economy, the use of income as a proxy for transaction would be less attractive. The same comment applies if exports are growing rapidly, because the level of transactions increases with the level of exports but exports are not a function of income.

The variation in the income distribution is important in the sample period for Uruguay, and exports are being promoted at present. These factors would have to be taken into consideration for a more precise specification of a transactions version of the demand for money function.

The type of measurement problem related to systematic deviations between measured and actual income does not seem to be the case for Uruguay. This is the case of recent experience in the U.S. A very reliable demand for money function began to perform poorly in the last years, and one of the possible explanations used was the amount of activities not reported that would make a much higher GNP than what it is actually measured.

The other measurement problem of importance is in the opportunity cost of holding money. The use of the rate of inflation as a proxy is common practice when the rate of inflation is as high as in the Uruguayan experience. But even with the low income growth and low investment, the real return on capital should have had some variation in a twenty year period. The interest in measuring this variable is also related to the fact that lower rates of inflation would produce less efficient and eventually no proxy at all for the opportunity cost. The trend towards the elimination of ceilings to nominal rates of interest will help in providing better measures of holding money in the future.

Estimation problems

Appendix A discusses the features of the nonlinear methods in estimation. In particular, the limitations of the numerical optimization processes show that finding the minimum of the sum of squares residuals is not a trivial question in nonlinear regression as it is in linear. The problem is also that the results of the optimization carry-over to the rest of the statistical work of producing tests and confidence intervals around the estimates. If the true minimum is not reached in the first place, the rest of the work is invalid.

In the particular case of this present work, the results of the full model are the most suspect of not reaching the absolute minimum of the least squares. This is an important limitation because if it were true it would invalidate the most general model presented here.

Other methods could be used to deal with nonlinear estimation, and sometimes more than one method could be used at the same time. This of course increases computational expense.

But, as it was also mentioned before, the probable reason for the poor performance of the full model and Submodel 1 is related to another type of estimation problem, the near singularity of the X'X matrix in the estimation of the parameters. This is a consequence of the existence of multicollinearity. As a result, the inversion of the matrix to calculate the coefficients is very slow and risky and the required minimization of the least squares residuals is not complete.

These results sometimes happen and of course they do not invalidate the results of the rest of the submodels.

Analysis of subperiods

The sample period was subdivided for just one of the submodels and not for a specific analysis of the effects of changes in the sample periods. This is an important limitation because of the characteristics of the sample period in Uruguay. Again, the limitation is computational expense and time, but the nonavailability of data for the period before 1957 had also an important bearing on the decision of considering just one sample period.

There is no doubt that the extension of the model is a crucial issue. Demand for money models in the U.S. involve frequently more than 50 years of data, and subperiod regressions are analyzed in detail. As a matter of fact, the question of stability of the demand for money has to be necessarily answered in the context of a long period with somehow different characteristics because otherwise it presents little analytical interest.

The definition of money

As it was discussed before, there is still an important polemic regarding the best definition of money. Many authors estimate models for more than one definition. In the Latin American studies discussed, time and savings deposits appear also in the definition of money. Deaver (4) argues that the existence of significant quantities of time deposits coexisting with strongly negative real interest rates is an evidence of the public's use of time deposits as money.

All of these questions were addressed before, and one of the conclusions had been that other quasi-money assets could be included as independent variables in the model, so that the changes in the public's portfolios would be taken into consideration.

In this work this approach has not been followed, so even when changes in the public's preferences regarding their portfolios and changes in the relative costs of holding the different close substitutes are considered relevant, their effects on the stock of currency and demand deposits are not explicitly considered.

This is also a limitation, but with the above methodology the inclusion of more independent arguments in the function would have been too expensive.

Seasonality in the data

Data for real cash balances are not deseasonalized, and that introduces another element of variation not explicitly accounted for in the regressions.

Seasonal dummy variables could have been used but again the same type of problem would have arisen, namely length of the model and computational expense.

Seasonal elements are present, and produce some important variations towards the end of the last quarter and beginning of the first. Its inclusion would improve the overall fit of the model.

The functional form problem

The variables were taken in natural logs so that the different coefficients estimated are elasticities. This is a standard procedure and implies an easier handling of the results obtained. Linear and log-linear forms have also been used in the past, some authors even provide more than one functional form in their studies.

As it was mentioned, Zarembka (30) presented a study in which, following the methodology by Box and Cox, both the linear and logarithmic forms of the demand for money function were a special case of a generalized functional form. He went on to estimate the demand for money and showed that in some definition, the results would be different with the use of either form. Also, the use of the power function approach permitted him to estimate a functional form that differed from both the linear and logarithmic form in a significant way.

In other words, the choice of the functional form is also an analytical problem. To estimate the correct functional form it is necessary to use a maximum likelihood approach to estimation. The likelihood function has to be maximized in terms of all the structural parameters in the model, the variance of the residuals and the power, or functional form parameter. To solve the simultaneous system involves computing the variance of the residuals for every value of the power. A number of regressions have to be produced to analyze the values of the likelihood function, and this makes the procedure cumbersome for more than one power.

The functional form study was attempted in this study but no definitive answer was reached before running out of computer time. The likelihood function was still increasing for values of the power below zero (which represents the logarithmic form, see the Zarembka study). Therefore, this fragmentary evidence points at the fact that the logarithmic form would be more appropriate than the linear in the worst situation, but both could be inappropriate. This is an unresolved question in this study, and future work could provide a better answer regarding the correct functional form.

Methodological Decisions that Affect the Selection of the Model

In this second group, the limitations are more important in the methodological sense.

Supply exogenous

Several authors of studies discussed above provide endogenous explanations of the supply of money. The importance of such a step is to give a more realistic view of the monetary sector. For the purpose at hand it was sufficient, apparently, to show it as exogenous. But as it will be discussed, it is a limitation that carries a varying degree of risk according to the variables that are actually affecting the evolution of the money supply.

The money stock (M) at a given time is defined as the product of the monetary source base (B) and the base multiplier (m).

$$M = m \cdot B$$

The monetary source base is composed of the commercial bank vault cash and the reserve cash at the Central Bank plus the currency outside the banking system. m is a ratio that considering the definition of money used in this work, equals currency plus demand deposits over currency plus reserve requirements.

$$m = \frac{C+D}{C+R} \text{ or } m = \frac{\frac{C}{D} + 1}{\frac{C}{D} + \frac{R}{D}}$$

Then, m depends on the currency-demand deposits ratio and on the reserves-demand deposits ratio.

Varying reserve ratios affect m, which is also affected by the relative preference to own demand deposits instead of other forms of quasi-money or other financial assets.

The reserve ratio in particular is normally assumed to respond to both institutional and behavioral factors. Central Bank fixes legal reserve requirements, but commercial banks are observed to hold excess reserves which are not necessarily proportional to legal requirements.

Diz (7) studies this "precautionary" reserve behavior for Argentina. He hypothesizes that its amount is a function of the demand for that type of reserves by the commercial banks and of the rate at which the Central Bank is injecting or withdrawing reserves. The demand on the part of the banks in turn depends on the opportunity cost of holding reserves and the composition of the public's deposits, since different deposits have different legal and also precautionary reserve requirements.

The important issue is that all these decisions depend partly on variables that also affect the public's demand for money. Changes in the composition of deposits in the banking system arise in part from changes in the public's preferences and expected opportunity costs. The expected change in the rate of inflation is one of the elements in the cost, the main one analyzed in this paper. But this in turn is one of the variables affecting the cost of holding reserves by the banks.

So, a variety of factors affect the base multiplier and therefore the stock of money at a given time. These elements are not necessarily exogenous in the sense that their effects have to be analyzed not only on the demand but also on the supply side.

Since the particular feature of the demand for money (and of the demand of any other asset) is nonobservability, making necessary the study of the equilibrium of the monetary sector, the approach used in this paper involves the consideration of the effects of one of the variables on just one side of the equation.

What may be still more important as a theoretical aspect not explicitly attacked in the model is directly the consideration of variations in the monetary source base.

There are two different assumptions that appear the most interesting in regard to the question of how the monetary base changes through time. One assumes that the base changes exogenously, or rather that every new stock is a function of the previous stock and a random variation. The other is that the rate of inflation has an important bearing on the rate at which money is being created in the economy.

Under this last assumption, the base is affected directly by expected changes in the rate of inflation, so supply should be "endogenous". Implicitly then, the other assumption was used in the paper. But the question of which of the two forms is more appropriate is found not to be independent of the way the public's expectations are modeled. This question is presented below.

The question of "rationality" of the adaptive expectation mechanism

In Uruguay as in many other countries, government deficits are primarily financed with the creation of money (increases in the base). The size of the monetary source base is continuously increased independent of the monetary authorities' desires. Theoretically they could sell bonds to decrease the public's liquidity and compensate the effect of government deficits. But there are limitations to this process in the size of the capital market, the reluctance to use indexed bonds in the presence of high inflation rates, etc. So, this practical possibility did not exist in the sample period, leaving then a certain proportionality between the size of the deficit and the rate of money creation in the economy. This fact could produce a link with expected inflation. The government could try to be a step ahead of the inflation process and obtain more resources by increasing faster the rate of money creation. This process has been widely discussed in the literature and is known as the "tax on cash balances" (see Cagan (1, and Deaver (4) for an application to Chile).

If the government is actually using this mechanism to be a step ahead of inflation, the result is that a "feed-back" is created from inflation to the money stock. Increases in the money stock have effects on the increase in the inflation rates, this in turn modifies expectations of future rates and tends to feed more money creation. Cagan in particular blames this process as one of the original factors for the hyperinflations he studied.

More recently, Sargent and Wallace (22) take a closer look at Cagan's work on hyperinflations. Their interest is to test the "rationality" of the adaptive expectations mechanism used by Cagan (also used in this study in a different form).

Sargent and Wallace argue that if the process described above actually occurred, then the use of an adaptive expectation process is "rational". "Rational" is taken in the general sense first used by Muth (21). He defined it as a process through which what the model forecasts or expects the rate of inflation to be is exactly what the public also expects it to be. If the process of expectation is not "rational", then it would mean that the public is believed to hold systematically different expectations from what the model is forecasting. This would be "irrational", because it would assume that the public never catches up with what the government is doing, never learn by experience.

Muth's approach could provide a check on the validity of the scheme of expectations assumed in the model. It is convenient to follow the discussion by Sargent and Wallace to see the relevant arguments.

The model used in this study was the following:

$$M_t^d = \alpha + \beta i_t^e + \gamma Y_t^e \qquad (1)$$

Expected income and the intercept term could be eliminated to simplify the exposition. Define:

$$n_t = \log \overline{M}_t - \log \overline{M}_{t-1}$$

Therefore n_t is the rate of increase in the money stock.

Equation (1) can be lagged once and then subtracted from itself at time t to obtain:

$$n_t = i_t + \beta(i_t^e - i_{t-1}^e) + e_t - e_{t-1} \qquad (21)$$

Equation (6) gave the adaptive expectations scheme for the rate of inflation:

$$i_t^e = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j i_{t-j} \quad \text{with} \quad j = 1,2,3, \ldots \qquad (6)$$

By substituting this equation in the above expression (21), one obtains a value for the actual rate of inflation that does not necessarily coincide with how the public is assumed to model expectations (Equation (6)).

Sargent and Wallace then go on to provide the framework to avoid this apparent "irrationality" in the construction of the model.

The approach they use is to start out assuming rational expectations on the part of the public, or:

$$i_t^e = E_t(i_{t+1}) \qquad (22)$$

where the symbol E denotes the mathematical expectation process.

For rationality to exist, what the public expects the rate of inflation to be has to coincide with what the model is forecasting mathematically speaking. By substituting (22) into (21) and rearranging:

$$i_t = n_t - \beta E_t(i_{t+1}) + \beta E_{t-1}(i_t) - e_t + e_{t-1} \qquad (23)$$

At time t+2, and always assuming that the expectations corresponds to a one time period horizon:

$$i_{t+1} = n_{t+1} - \beta E_{t+1}(i_{t+2}) + \beta E_t(i_{t+1}) - e_{t+1} + e_t \qquad (24)$$

By taking expectation operations on (23) and (24), rearranging and operating, one arrives at an expression of the rate of inflation as a function of infinite sums of values of the rate of increase in the money stock n_t and future random shocks in the demand for money function (represented by the errors e_t). Current values of the endogenous variables of the model depend on the public's expectations of future values of "exogenous" variables like the rate of money creation in the economy. Equations (23) and (24) express this process clearly. The rate of inflation at t depends on the expected rate of inflation next year which in turn (Equation (24)) depends on the expectations in t+2, etc.

To complete the mechanism, the model has to provide an assumed mechanism of how the money stock is increased and relate the public to it. The public can be assumed to ignore how money is created in the economy, so that this would mean "irrational" expectations are playing a role. If on the contrary, the public will be assumed to understand the process through which money is issued, then their expectations will be "rational".

Sargent and Wallace (22) go on to state that two different assumptions could be formulated regarding the process of money creation. They were mentioned before. One is the exogenous (no feedback of inflation on money creation):

$$n_t = \sum_{j=0}^{\infty} w_j n_{t-j} + \bar{e}_t \tag{25}$$

in which $\bar{e}_t$ is a nonautocorrelated, stochastic term. In the formation of the money supply decision, future rates of inflation have no effect, only past changes in money supply and random effects are the explanatory factors.

If it is assumed that the public also knows Equation (26), then the rate of inflation i_t can be written as a geometrically mean of past increases in the money supply, and a regression equation set up. There is a direction of causation here, money creation affects inflation but not vice-versa. The opposite, running a regression to explain the money stock as a geometrical mean of past rates of inflation would not be "rational" given the actual process of money supply creation. In other words, using Equation (25) when the government is actually financing expenditures by resorting to money creation is then identical to assuming "irrationality" on the part of the public.

The other assumed behavior of the money supply is when the government is actually using the expected rates of inflation to create money, Equation (25) is no longer valid and instead:

$$n_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j i_t \tag{26}$$

and if the public also believes this is the actual process, then Sargent and Wallace show that with this assumption the expression for the expected rate of inflation turns out to be:

$$E(i_t) = i_t^e = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j i_{t-j}$$

which is Equation (6), the adaptive expectations process.

The conclusion is then that the adaptive expectations approach is "rational" when there is feedback from inflation to the rate of money creation.

This discussion is important because subordinates the form of the regression actually used to a previous definition of the "direction of causation" of the corresponding variables. In this context, the use of the money stock, the income velocity or the rate of inflation as dependent variables is no longer the same, since a direction of causation problem is involved. The reason for this is that the money supply may or may not be truly exogenous.

The test for the direction of causation was not done previous to this study, therefore the approach followed constitutes a limitation. The possibility that the expectation formation mechanism is "ad-hoc" one is real and not expressly answered.

The simultaneous equation problem

For a body of theory, the consideration of an isolated monetary equilibrium is a methodological simplification that strips away the complex interrelationships actually present in the economy. The idea is that the actual process of mutual interdependence is the most important issue, and therefore any model which could account for most of these interrelationships would have theoretical advantage over a single equation model. This kind of model would have to include separate equations for the different relations and involve a simultaneous estimation. This approach would argue that a single equation model has little use for policy-makers.

This traditional view has been challenged in recent years by the success of a single equation model, the St. Louis model, in explaining the rate of inflation when compared with the alternative large simultaneous models of the U.S. economy. It also seems more advantageous for the policy-maker to keep in mind a single equation model than a complex system of interrelationships.

This is however, an unresolved question. There is no doubt that a simultaneous model in several equations could give more insight into the cost of stabilization processes and the relationship between inflation and growth, but the effort could probably be too large given the present state of available information in many underdeveloped countries.

CHAPTER VI. SUMMARY AND CONCLUSIONS

The demand for money is an unobservable process. To be able to identify it, the existing money stock has to be used as the main point of reference. Given the assumption of stability of demand, a regression with money stock as the dependent variable is applicable.

To formulate the demand for money function some assumption regarding the formation of expectations is necessary. The adaptive expectation scheme has been widely used in the past and in particular this method is proven to be "rational" when the rate of inflation affects money creation and not vice-versa. The test for this assumption is not provided in this study so the expectation mechanism used is ad-hoc.

The demand for money function in Uruguay appears to be strongly related to the expected rate of inflation. The demand for money is highly elastic with respect to inflation cost and the average period of expectation formation was found to be over 4 years.

Income elasticities are found to be low in this study. Permanent income elasticity is positive but very close to zero, current income elasticity is negative. This finding is not reliable, however, due to the little variation in real income in the sample period.

Of all the other possible arguments in the formulation, an income distribution index was included. It turned out to have the right sign but no statistical value in the explanation. This result is also limited because of the way that the income distribution index was defined and the short time period used in the analysis.

Other variables could possibly be successful. Among them, changes in portfolios or other proxy variables for the level of transactions in the economy.

Seasonality factors were not excluded a priori from the data, and they would also normally account for part of the variation in real cash balances.

The polemic regarding the proper theoretical approach of the demand for money received an inconclusive answer in the study, as was expected. Even the question of stability has to be related to the particular sample period used, there exists the possibility of wide fluctuations in the cost elasticity of the demand for money due to the high values obtained.

The question of the use of the equation for policy also remains a sensitive problem. Even when the stability in the period would suggest the use of a money supply type of policy to control inflation, given the recurrence of the government to money creation to finance expenditures the cut would in practice be directed to the private credit. The effects on growth and unemployment could not possibly be evaluated without a broader model in which the main interrelationships among the rate of inflation, the money stock, the level of unemployment, investment and income in the economy are included.

The results of the present model produce also a warning for the potential policy-maker in the sense that the public seem to adapt their expectations in a slow form. This has consequences on the extension of time that a tight money supply policy would have to be maintained to become effective. This is, of course, relevant if the adaptive expectations scheme is close to the public's behavior in reality. Cagan (1) shows that the announcement of currency reform at the last stages in hyperinflation produces fast departures of the mechanisms of formation in expectations. In the context of high rates of inflation, the same effect could be obtained with wage and price freeze policies.

What remains as an important consideration is that there exists the possibility of "threshold" effects. The decrease in the rate of inflation that could eventually be obtained through different policies could determine that the demand for money as a function of the expected rate of inflation could vanish.

It also remains to be seen if the partial control in the money supply that the continuing government deficits imply can be effective in the decrease of the rate of inflation.

If this is not the case, the long period of tight credit conditions for the private sector could mean a slower rate of growth than the potential. In other words, the size of the government deficit is really the true exogenous variable, and its control should probably be the main target.

LIST OF REFERENCES

1. Cagan, Phillip. "The monetary dynamics of hyperinflation". In Milton Friedman ed., Studies in the Quantity Theory of Money. Chicago: University of Chicago Press, 1956.

2. Chow, Gregory. "On the long run and short run demand for money". Journal of Political Economy 74 (1966):111-131.

3. Corbo-Lioi, Victor. Inflation in Developing Countries, An Econometric Study of Chilean Inflation. The Netherlands: North-Holland, 1974.

4. Deaver, John. "The Chilean inflation and the demand for money." In David Meiselman ed., Varieties of Monetary Experience. Chicago: The University of Chicago Press, 1970.

5. Diaz-Alejandro, C. F. Exchange rate Devaluation in a Semi-industrialized Country: the experience of Argentina 1955-1961. Cambridge: The M.I.T. Press, 1965.

6. Direccion General de Estadistica y Censos (DGEyC). Indice Medio de Salarios. June 1977.

7. Diz, A. C. "Money and prices in Argentina, 1935-1962." In David Meiselman ed., Varieties of Monetary Experience. Chicago: The University of Chicago Press: 1970.

8. Feige, Edgar. "Expectations and adjustments in the monetary sector". American Economic Review 57 (1967):462-473.

9. Feige, Edgar. The Demand for Liquid Assets: A Temporal Cross Section Analysis. Englewood Cliffs, N.J.: Prentice-Hall, 1964.

10. Feige, E. and Parkin, M. "The optimal quantity of money, bonds, commodity inventories and capital". American Economic Review 3 (1971):335-349.

11. Friedman, Milton. "A theoretical framework for monetary analysis". Milton Friedman's Monetary Framework. In Robert Gordon, ed., Chicago: The University of Chicago Press, 1974.

12. Friedman, Milton. "The Quantity Theory of Money - A Restatement". In Milton Friedman ed., Studies in the Quantity Theory of Money. Chicago: University of Chicago Press, 1956.

13. Fuller, Wayne and Martin, J. "The effects of auto-correlated errors on the statistical estimation of distributed lag models". Journal of Farm Economics 43 (1961):71-82.

14. Goldfeld, Samuel. "The case of the missing money". Brookings Papers on Economic Activity 3 (1977):683-740.

15. Goldfeld, S. and Quandt, R. Non-linear methods in Econometrics. Amsterdam: North-Holland, 1972.

16. Harberger, Arnold. "The Dynamics of inflation in Chile". In C. Christ ed., Measurement in Economics, Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld. Stanford, Ca.: Stanford University Press, 1963.

17. International Monetary Fund. International Financial Statistics, Volume XXXI, Number 5 (1978).

18. Johnston, John. Econometric Methods. New York: McGraw-Hill, 1972.

19. Meyer, P. A. and Neri, J. A. "A Keynes-Friedman money demand function". American Economic Review 4 (1975): 610-623.

20. Muth, John. "Optimal properties of exponentially weighed forecasts". Journal of The American Statistical Association LV (1960):299-306.

21. Muth, John. "Rational expectations and the theory of price movements". Econometrica XXIX (1961):315-335.

22. Sargent, T. and Wallace, N. "Rational expectations and the dynamics of hyperinflation". International Economic Review 2 (1973):328-350.

23. Sims, Christopher. "Money, income and causality". American Economic Review 4 (1972):540-552.

24. Starleaf, Dennis. "The specification of money demand-supply models which involve the use of distributed lags". Journal of Finance 4 (1970):743-760.

25. Theil, Henri. Principles of Econometrics. New York: John Wiley and Sons, 1971.

26. Tsiang, S. C. "The monetary theoretic foundation of the modern monetary approach to the balance of payments". Oxford Economic Papers 29 (1977):319-338.

27. Tsiang, S. C. "Walras' Law, Say's Law and Liquidity preference in general equilibrium analysis". International Economic Review 3 (1966):329-344.

28. United Nations. Monthly Bulletin of Statistics. Volumes XI-XXI (1957-1967).

29. Vogel, Robert. "The dynamics of inflation in Latin-America, 1950-1969". American Economic Review 1 (1974):102-114.

30. Zarembka, Paul. "Functional form in the demand for money". Journal of the American Statistical Association 63 (1968):423-443.

APPENDIX A: STATISTICAL METHODS

Nonlinear Estimation Procedures

The problems that nonlinearities produce in regression could be subdivided into two groups: (a) numerical optimization problems, (b) variations in the traditional properties of the estimators.

In the traditional linear form of regression there are no fundamental optimization problems. Given a representation of the general regression model like the following:

$$y = f(x,\theta) + u$$

where

x = nonstochastic vector of k components

θ = vector of p components representing the parameters

y = vector with the observations of the dependent variable

u = vector of stochastic terms

Estimates are obtained by minimization of the objective function:

$$S(\theta) = (y_i - f(x^i,\theta))^2$$

This process generates $\overline{\theta}$ as estimated values of the parameters, being $S(\theta)$ the minimum of the objective function. Differentiating $S(\theta)$:

$$\frac{\partial S}{\partial \theta_j} = 2 \sum_{i=1}^{n} (y_i - f(x^i, \theta)) \frac{f(x^i, \theta)}{\theta_j}) = 0 \qquad j = 1, \ldots p$$

If the function f is linear in the p parameters, the partial derivatives inside the parentheses do not involve θ. Therefore, the set of p first order conditions, the "normal equations", will also be linear.

On the other hand, if f is nonlinear those partial derivatives will also include θ, and when they are multiplied by f more nonlinear terms come up. The conclusion is then that those normal equations will not have a simple analytical solution.

In other words, when the function is linear the expression for the sum of squares residuals is quadratic. Local extrema in this function will also be global extrema. When nonlinearities are present, some methods have to be used to separate the different local extrema from the global.

Numerical methods are used with this problem. In general these methods consist of choosing a departing value of the function and then iterate towards the optimum with a scheme like the following:

$$x^{p+1} = x^p + h^p D^p$$

where

x^p = approximation of the i^{th} iteration

D^p = direction vector

h^p = positive scalar

The iterative procedure continued until a predetermined convergence criterion is reached. Different methods provide alternative selections for h^p, D^p and the convergence criterion. Methods will differ in terms of degree of accuracy in the location of stationary points, speed of convergence, cost in computer time, human effort, etc. They will sometimes produce a different solution which constitutes a real problem.

There are several numerical methods currently used, the most common the Steepest Ascent, Marquardt, Quadratic-Hill climbing, Newton, and modified versions. The discussion of the relative merits of each will not be done here (see for example Goldfeld and Quandt (15)).

The estimations in this present work were done with the Gauss-Newton method available in the SAS computer package. Its procedure is to "linearize" the equation by expanding it around a given estimate of θ, which is arbitrary, using a Taylor series expansion:

$$y-f(x,\theta^*) = \sum_{t=1}^{\infty} \frac{\partial f(x,\theta^*)}{\partial \theta_i}(\theta_i-\theta_i^*)$$

where

θ^* = arbitrary starting point

Calling θ^t the estimate of θ at the start of the t iteration, the above expression can be written as:

$$Y-F^t = Z^t D^t + v$$

where

Y = vector of the y's

F^t = vector of predicted values at t

Z^t = matrix of partial derivatives of F respect the parameters

$D^t = \theta-\theta^t$

v = vector of error terms

D^t can be obtained from the above using linear theory:

$$D^t = (Z'^t Z^t)^{-1} Z'^t (Y-F^t) = \theta-\theta^t$$

The new estimate of θ to perform the next iteration is simply obtained from this last expression:

$$\theta^{t+1} = D^t + \theta^t$$

This new value can be in turn used to relinearize $f(x,\theta)$ continuing with the iteration procedure. The search ends when D^t is sufficiently close to θ. The method could possibly not converge, but if it converges it will satisfy the first order conditions for a minimum. Convergence means:

$$0 = D^t = (Z'^t Z^t)^{-1} Z'^t (Y - F^t)$$

which implies

$$Z'^t (Y - F^t) = 0$$

This is the same condition which results from differentiating the sum of squares residuals with respect to all the parameters and setting the result equal to zero. Thus, if the process converges at all the resulting point satisfies first order conditions for a minimum. However, it could be a local rather than a global minimum, as discussed above.

In the following discussion it will be assumed that a minimum has been reached. The second problem then arises, what are the new properties of the estimators and the significance of the test and confidence intervals that can be constructed when compared with those in linear regression theory.

In linear models, the construction of confidence intervals runs as follows. Given the model:

$$Y = X\theta + U$$

A partition can be done on the θ's complementing the X's to make the matrices conformable:

$$Y = X_1\theta_1 + X_2\theta_2 + U$$

where θ_1 contains (k-q) elements and θ_2 contains q elements. Performing a test on the explanatory power of the θ_2 parameters simultaneously requires calculating the reduction in the sum of squares resulting from the substitution for the least squares values of the θ's. The formula for the least squares estimates of the parameters is:

$$\hat{\theta} = (X'X)^{-1}X'Y$$

For the partitioned set the estimates of θ_1 and θ_2 are rather cumbersome expressions, but it can be shown that in almost all cases in which the variables are not independent, the expressions for the estimates of each set of parameters depend on the sample values and on the true values of the coefficients in the other set.

Then, if for example the elements of θ_2 were set at fixed values, least squares estimates of θ_1 could be derived as functions of θ_2:

$$\hat{\theta}_1(\theta_2) = (X_1'X_1)^{-1}X_1'(Y-X_2\theta_2)$$

For the whole set of parameters θ, the sum of squares residuals could be shown to be:

$$S(\hat{\theta}) = S(\theta_1,\theta_2) = U'(I-X(X'X)^{-1}X')U$$

where I is the identity matrix, U the error vector and U' its transposed.

On the other hand, the sum of squares corresponding to the θ_2 subset of parameters is composed of the direct effect plus the indirect through the influence on the other θ_1. Then:

$$S(\hat{\theta}(\theta_2),\theta_2) = U'(I-X'(X_1'X_1)^{-1}X_1')U$$

could also be obtained.

The quantity:

$$\frac{(S(\hat{\theta}_1(\theta_2),\theta_2) - S(\hat{\theta}_1,\hat{\theta}_2))/q}{S(\hat{\theta}_1,\hat{\theta}_2)/n-k}$$

has the F distribution with (q, n-k) degrees of freedom.

The substitution in this expression of a hypothesized value of θ_2, say θ_2^* can be evaluated by comparing the value of the expression with the corresponding F value at the preset level of confidence. If the critical value is greater then θ_2^* is rejected. Confidence intervals are obtained by equating the expression to the F value at the level of confidence and rearranging. The resulting confidence contour will be also at the 1 percent level of confidence.

An important theorem then states that this confidence region is ellipsoidal in the space of θ_2 with center at θ_2. It can also be shown that an identical expression for the region would be:

$$(\hat{\theta}_2-\theta_2)'V(\hat{\theta}_2-\theta_2) = \frac{q}{n-k} S(\hat{\theta})F(q,n-k)$$

where

$$V = X_2'X_2 - (X_2'X_1)(X_1'X_1)^{-1}(X_1'X_2)$$

and the expression above meets all the requirements to be the representation of an ellipsoid centered on θ_2.

In consequence, in the case of linearity the confidence region is bounded and in particular has an ellipsoid form.

In a nonlinear model of the form stated before, the joint confidence contours are expressed as follows:

$$\frac{(\hat{D}-D)'Z'Z(\hat{D}-D)/p}{S(\hat{D})/(n-p)} = F(p,\ n-p)$$

using the notation of the first part. In the above, $S(\hat{D})$ stands for:

$$S(\hat{D}) = (Y-F(X,\hat{\theta})-Z\hat{D})'(Y-F(x,\hat{\theta})-Z\hat{D})$$

which is just the sum of squares residuals of the previously used form:

$$Y-F^t = Z^tD^t + U$$

When the point of minimum is reached, D = 0, then the confidence contours is given by:

$$\frac{(\hat{\theta}-\theta)'Z'Z(\hat{\theta}-\theta)/p}{S(\hat{\theta})/(n-p)} = F(p,\ n-p)$$

since $D = \theta - \hat{\theta}$

Then, this expression states that the form of the contours is ellipsoidal around the estimated θ's. This is one of the methods to produce confidence intervals, and it could be shown that other methods produce different answers.

In the above, the assumption of normality was used implicitly because the method of construction of the contours was by way of comparing with the linear case. It is a somewhat restricted approach because of the recurrence to the linear models background.

A more complete and general approach is used with the maximum likelihood method. It will permit a departure from the normality assumption of the distribution of the error terms and generate asymptotic results of great interest.

The above method of obtaining confidence intervals depends on the results of the numerical optimization process. If the process fails to give the true global minimum then the last linearization will be meaningless, and so will be the confidence contours built around the estimates. As a general principle it could be stated that least squares estimates possess no general properties in nonlinear models, as they do in linear when the distribution of the residuals is normal.

Setting the likelihood function:

$$L(X/\theta) = \prod_{i=1}^{n} f(X^i, \theta)$$

Assuming normality, the maximization of this function produces the same estimates of the parameters of the least squares procedures. The latter will then share all the properties of the maximum likelihood estimates. In particular, the confidence contours discussed before are also isolikelihood contours with this approach.

However, further results can be achieved by using the properties of the likelihood function.

One of the basic results of the theory in this field is the existence of a lower bound for the variance of an unbiased estimator g of a function g. This is called the Cramer-Rao bound:

$$\text{Var } \hat{g} \geq \frac{-(g'(\theta))^2}{E(\frac{\partial^2 \log L}{\partial \theta^2})}$$

The right hand side is the minimum variance bound (MVB). It could be showed that when an MVB estimate exists, it does so for only one function $g(\theta)$. This result is proven to be valid only asymptotically.

Given the special case $g(\theta) = 0$; $g'(\theta) = 1$ the following applies:

$$\mathrm{Var}(\hat{\theta}) \geq - \frac{1}{E(\frac{\partial^2 \log L(X/\theta)}{\partial \theta^2})} = R^{-1}(\theta)$$

If $\hat{\theta}$ is a maximum likelihood estimator of a vector parameter θ, then $\hat{\theta}$ will be asymptotically distributed $N(\theta, R^{-1}(\theta))$.

Writing the asymptotic joint normal density of $\hat{\theta}$

$$f(\hat{\theta}) = (2\pi)^{-p/2} /\Sigma/^{-1/2} \exp(-\tfrac{1}{2}(\hat{\theta}-\theta)'\Sigma^{-1}(\hat{\theta}-\theta))$$

The exponent of this expression follows a χ^2 distribution, therefore confidence intervals can be readily constructed. When the number of degrees of freedom tends to infinity this result will approach the one obtained before with the linear approximation. This result proves that the reliance on a very large sample when using nonlinear squares methods produces estimates which are asymptotically approaching those of the maximum likelihood approach. The properties shared asymptotically would then be: (a) if a maximum likelihood estimate exists (the relevant derivatives of the likelihood function exist) then the estimate of the vector θ is consistent; (b) there is no more than one root of the first order conditions for a maximum of the likelihood function that is a consistent estimator of θ; (c) it is normally asymptotically distributed; (d) it is efficient (attains the Cramer-Rao bound); (e) it is sufficient if $f(x,\theta)$ admits a sufficient result.

In conclusion, the numerical optimization methods do not allow a complete confidence in reaching these desirable properties. This is so because the global minimum can't be obtained with complete certainty.

If the true global minimum of the least squares is assumed to be obtained in the models of this present study, then the size of the sample used determines that the asymptotically properties stated apply.

Serial Correlation in the Presence of Lagged Dependent Variables

The problem of serial correlation has to be addressed specifically since the methods discussed before to deal with the nonlinear situation assume that the last linearization produces an equation in which ordinary regression could be applied. But the existence of lagged dependent variables and serial correlation violates two assumptions of ordinary least squares.

First, when lagged dependent variables are present the assumption of independence of the error term with the vector of independent variables is not valid any more. Also, the serial correlation of the residuals violates the independence assumption regarding their behavior.

Given:

$$y = XB + e$$

where

$$X = (X_r X_f)$$

with

X_r = lagged values of y

X_f = the rest of the independent variables

Assuming that e is the vector with zero mean, finite variance and with no serial correlation, and that the lagged values of the dependent variable are taken as given (nonstochastic), then the estimators:

$$\hat{B} = (X'X)^{-1}X'y \qquad s^2 = \frac{e'e}{n-k}$$

where

n = sample size

k = number of parameters

These estimators are consistent for B and σ^2 respectively.

A number of asymptotic results also follow. In other words, the mere presence of lagged dependent variables when no serial correlation is present does not constitute a major problem and it can be handled in the usual way.

However, in the presence of autocorrelated disturbances even the reliance on large samples will not suffice to produce consistent estimators. Assume:

$$e_t = \rho . e_{t-1} + u_t$$

is a first order autocorrelation process for the e's, with u_t another stochastic term with zero mean, finite variance and nonserially correlated. It is possible to show that:

$$\operatorname*{plim}_{n \to \infty} \hat{B} = \frac{B+\rho}{1+B.\rho}$$

where plim denotes the probability limit.

According to this result, if $\rho \neq 0$ which is the case when serial correlation is present, the estimated parameters will not be consistent. The explanation for this result lies in the fact that with lagged dependent on the right side of the equation, and with e_t correlated with e_{t-1}, e_t will also be correlated with y_{t-1} which in turn is in the independent side of the y_t formulation. The least squares assumption that the error term is independent respect to the independent arguments of the equation cannot be sustained any longer. On the other hand, if there were no lagged dependent variables in the equation the estimators $\hat{B}$ are proven to be consistent. But this is not the case of the models discussed in this paper.

The presence of autocorrelated disturbances produces other undesirable effects which are common to models with and without lagged dependent variables. The sampling variances of the regression coefficients will be underestimated, therefore affecting the t and F tests which will not normally have the previous precise form, and also producing inefficient predictions. The particular effect of the distortion is varied, but it might be even more important in the case of lagged dependent variables.

In ordinary least squares, the parameters are estimated by means of:

$$\hat{B} = (X'X)^{-1}X'y$$

Which, upon substitution of the vector y could be written:

$$\hat{B} = b + (X'X)^{-1}e$$

so that

$$E(\hat{B}) = B$$

since by assumption $E(e) = 0$. Next, the covariance matrix in this case is:

$$Var(\hat{B}) = \sigma_e^2(X'X)^{-1}$$

Assuming the existence of autocorrelation in the model but no lagged dependent variables for simplicity, this same variance will be represented by:

$$\begin{aligned} Var(\hat{B}) &= E((\hat{B}-B)(\hat{B}-B)') = \\ &= E((X'X)^{-1}X'ee'X(X'X)^{-1}) \\ &= (X'X)^{-1}X'VX(X'X)^{-1} \end{aligned}$$

Where V represents a square matrix (nxn) of the form:

$$V = E(ee') = \begin{bmatrix} 1 & p & p^2 & p^3 & \cdots & p^{n-1} \\ p & 1 & p & p^2 & \cdots & p^{n-2} \\ p^2 & p & 1 & p & \cdots & p^{n-3} \\ \cdot & & & & & \cdot \\ p^{n-1} & p^{n-2} & p^{n-3} & & & 1 \end{bmatrix}$$

V substitutes the standard assumption of independent error terms currently represented by the following (in matrix form):

$$E(ee') = \sigma^2 I$$

Where I is the identity matrix.

The above formulation of the variances in both cases shows how it is underestimated when there are auto-correlated disturbances but the researcher uses standard regression theory to estimate the parameters (see Johnston (18) for an example).

The same type of explanation applies to the calculation of the estimator of the variance of the least square residuals, since:

$$E(ee') = (n-1)\sigma_e^2$$

is no longer valid.

Johnston (18) states that the bias introduced is not likely to be too important in the case of approximate randomness in the independent variables. But this is not the case when lagged dependent variables are present in the model. This is clearly seen when the expression for the estimated covariance of the residuals is written out:

$$E(e'e) = \sigma_e^2 \left(n - \left(1 + 2\rho \cdot \frac{\sum_{i=1}^{n-1} x_i x_{i+1}}{\sum_{i=1}^{n} x_i^2} + 2\rho^2 \frac{\sum_{i=1}^{n-2} x_i x_{i+2}}{\sum_{i=1}^{n} x_i^2} + \ldots + 2\rho^{n-1} \frac{x_1 x_n}{\sum_{i=1}^{n} x_i^2}\right)\right)$$

In the case of nonzero ρ the correlation between x_i and x_{i+1} will make the inside parentheses greater in absolute value, making the underestimation of the true variance still greater. The presence of lagged dependent variables assures a high correlation between part of the components of the X vector and is likely to produce this undesirable result.

The consequences on the t and F tests will not be discussed here, but are dependent on the above discussions of the effects on the sampling variances of the estimators and residuals.

All of the above discussion seeks to stress the basic question that the presence of serial correlation in models with lagged dependent variables produces invalid econometric work unless some degree of correction is explicitly introduced.

It is also important to remember the poor result that measurements of serial correlation through the Durbin-Watson and related statistics tend to have in models with lagged dependent variables. These types of measurements are biased towards their ideal values in this case, see Johnston (18), so their use has been completely avoided in this paper.

The standard procedure to correct for serial correlation is by means of a transformation of the original variables of the regression to other variables assumed free of serial correlation. This was the procedure used in this paper. Equation (14) represented a first order serial correlation scheme. By substitution in the model, the new formulation is supposed free of serial correlation because the new error term is assumed to have all the desirable properties of standard regression theory.

If serial correlation is eliminated by this transformation, then the previous discussion permits the use of regression to determine the estimates which will be consistent in large samples. However, the degree of serial correlation is crucial because the assumed first order scheme might not approximate the actual process. The use of a first order scheme is common practice because it is observed to correct most of the serial correlation and because higher orders are more expensive to compute. For every degree of serial correlation assumed in the model there is to include one more reduced form estimate for every variable in the model.

In this paper just a first order scheme was used, and the results are limited for this reason to the case in which most of the serial correlation can be corrected with such a scheme.

An alternative method exists in the case of linear model in the structural parameters, represented in this paper by one of the final submodels used. The method relies on the use of computer packages like SAS and consists in successive corrections for serial correlation using a first order scheme in each step and the estimated correlation coefficients of the disturbances in the previous step. In other words, every set of transformed variables is retransformed using the new correlation coefficient of the disturbances. If the correction is almost complete with the first step, then the assumption of the first order scheme accounts for most of the serial correlation. Typically, every step will produce new estimated correlation coefficients smaller and closer to the ideal value of zero. The method can be stopped at a desired preset low value of $\hat{\rho}$.

In the use of the method it was observed that the t value of the lagged dependent variable decreased systematically and also did the F ratio and the R^2 statistics. This serves as a strong warning to many researchers that in the past remain content with results of models with serial correlation. High t values and R^2 are meaningless in this context or at least much less important to determine the validity of the model than in the usual conditions. This also accounts for the fact that the discussion of the results in the nonlinear versions of the model did not focus the R^2 and F values obtained. The presence of lagged dependent variables assures high levels in both, but the partial correction for autocorrelation applied does not permit a wider use of those statistics.

APPENDIX B: THE DATA UTILIZED IN THE STUDY

The basic source of the data utilized in this study is the International Financial Statistics (IFS), a publication of the International Monetary Fund (IMF).

In turn, their original source is generally the Central Bank of Uruguay through its various bulletins and publications.

The availability of reliable data was the main factor in the definition of the sample period. There was no GDP data previous to 1957 reported in the IFS, and data directly from Uruguay did not arrive in time for this study.

Although this problem would not affect the submodels that do not include GDP in the study, there were also some major changes in the presentation of the accounts around this period. Currency and demand deposits in particular are not end of period figures like they were after 1957. Therefore, the construction of an end of period would not be possible for the previous period, and the use of averages for the period afterwards would also be difficult for lack of information. These facts made it advisable to start the sample period in 1957 and therefore obtain a uniform set of data. The resulting number of observations (80) seemed sufficient for the large sample requirements of the statistical methods used.

The construction of the money variable was done following the criterion used by the IFS publication for Uruguay. Currency outside banks is the first component, and also demand deposits at commercial banks. The deposits at one of the official banks, the Banco de la Republica are also included in spite of the partial character of monetary authority this bank possesses (total responsibility before 1966, at present it shares this function with the Central Bank, created in 1967). The Banco de la Republica performs important activities as a commercial bank also, so its demand deposits should also be included.

Another component of money is demand deposits by official corporations in the banking system. This is sensible given the extent of the government economic activities. Since in other countries the same activities would be carried forward by private firms, their demand deposits would be already included.

Other banks in the official banking system also receive demand deposits from the public and other official institutions. These are the Banco Hipotecario (State Mortgage Bank) and other smaller institutions of the savings and loans type. The deposits in these banks have not been included in the total of demand deposits because it has been traditional not to do so. At any rate, the proportion of demand deposits of these entities in the mass of their activities and in the total of the demand deposits at the commercial banking system is very small.

Only one price index was used in this study, the consumer price index (CPI). The reason is its traditional properties and it seemed more reliable than the alternative wholesale price index. The issue is not really important because of the narrow divergence they present in most of the period. Also, given the high rates of inflation, the relative divergence in the evolution of the indexes tends to be small relative to the absolute variation they present.

Since the IFS presented the CPI data as an average for the quarter, a search was done to obtain end of period data. Finally, the United Nations Statistical Bulletin was used for the first years, complemented by direct sources from the Statistics and Census Office in Uruguay for later years. The data match since the original source is the same.

The income variable selected was defined as GDP and obtained from the IFS already in real terms. It is traditional for studies in Latin America to use GDP instead of GNP. The difference between the two concepts is net factor payments from abroad and they are usually negative in underdeveloped countries. By eliminating this net transfer, the resulting quantity is a better proxy for domestic factors income. Of course, there is the problem of depreciation, subsidies and indirect taxes that separate GDP from an actual domestic income concept, but these complications were not considered of sufficient importance to avoid representing what is loosely called "income" in this study by GDP.

One important problem with GDP was the lack of quarterly data. Data were annual and centered in the middle of the year, so a procedure had to be used to make it compatible with the price and money data, which were end of period. The procedure was to assume the GDP data as end of second quarter and proceed to linearly interpolate between each two consecutive observations to produce data for every quarter. This procedure of course presents its limitations but was used as a first approximation, other authors have used it in the past and there is no other simple alternative available. Again, the size of the error is a function of the absolute variation in GDP. But as it was mentioned, income growth was small in the period.

The variables were taken in per capita form. Data was available to construct a population variable using the Census of 1963 and 1975. However, different assumptions were made regarding rates of population growth. The latter was divided in several parts. From 1957 to 1963, the assumed rate was 1 percent annually, and it carried forward until 1968. From 1968 to 1971 the rate was assumed to be lower, .5 percent. For the remaining years of the period the population was assumed roughly constant, and this would match the findings of the 1975 Census.

All the series utilized in this study were transformed into index for computational reasons, and are presented in Table 7.

Table 7. Data utilized in the study[a]

Date	Real GDP Index[b]	CPI[b]	Nominal Money Stock	Population Index	Real Wages Index[c]
1957					
I	101.00	25.00	912	93.90	
II	101.79	27.71	927	94.15	
III	100.88	27.89	906	94.38	
IV	99.97	29.17	928	94.62	
1958					
I	99.06	30.07	1,032	94.86	
II	98.12	31.15	1,092	95.10	
III	97.43	32.60	1,100	95.34	
IV	96.75	34.96	1,175	95.58	
1959					
I	96.07	40.03	1,311	95.82	
II	95.38	44.38	1,397	96.06	
III	96.24	48.18	1,486	96.30	
IV	97.11	51.81	1,662	96.54	
1960					
I	97.97	54.52	1,839	96.78	
II	98.83	59.23	2,067	97.03	
III	99.57	68.11	2,287	97.27	
IV	100.31	70.65	2,371	97.51	

[a]Sources: International Monetary Fund (17), United Nations, Monthly Bulletin of Statistics (28), and Direccion General de Estadistica y Census (DGEyC)(6).

[b]Base 1963.

[c]Base 1968.

Table 7 (Continued)

Year	Real GDP[b] Index	CPI[b]	Nominal Money Stock	Population Index	Real Wages[c] Index
1961					
I	101.05	71.92	2,498	97.76	
II	101.79	75.00	2,607	98.01	
III	101.22	77.17	2,618	98.25	
IV	100.65	77.89	2,774	98.50	
1962					
I	100.08	80.13	2,918	98.75	
II	99.51	81.71	2,918	99.00	
III	99.63	85.51	2,861	99.24	
IV	99.75	86.46	2,816	99.50	
1963					
I	99.88	89.00	2,922	99.74	
II	100.00	97.55	3,002	100.00	
III	100.42	107.70	3,061	100.25	
IV	100.84	124.00	3,391	100.50	
1964					
I	101.26	132.00	3,792	100.75	
II	101.69	140.00	3,929	101.00	
III	101.94	153.00	4,035	101.25	
IV	102.20	168.00	4,665	101.50	
1965					
I	102.45	188.00	5,496	101.76	
II	102.71	210.00	6,476	102.00	
III	103.65	250.00	7,802	102.25	
IV	104.59	315.00	9,483	102.51	
1966					
I	105.53	341.00	11,543	102.77	
II	106.47	391.00	12,766	103.00	
III	105.44	417.00	13,269	103.26	
IV	103.37	471.00	14,154	103.50	
1967					
I	103.37	582.00	15,252	103.75	
II	102.35	643.00	16,597	104.00	
III	102.65	864.00	18,869	104.25	
IV	102.94	1,111.00	25,650	104.50	

Table 7 (Continued)

Year	Real GDP[b] Index	CPI[b]	Nominal Money Stock	Population Index	Real Wages[c] Index
1968					
I	103.23	1,440.00	31,892	104.75	101.19
II	103.52	1,810.00	34,219	105.00	85.87
III	105.14	1,781.00	37,130	105.13	110.70
IV	106.76	1,848.00	43,095	105.26	108.45
1969					
I	108.38	1,931.00	51,764	105.39	111.24
II	110.00	1,993.00	56,883	105.52	112.49
III	111.32	2,045.00	63,608	105.65	109.70
IV	112.64	2,116.00	73,057	105.78	113.62
1970					
I	113.96	2,238,00	77,609	105.92	113.05
II	115.29	2,323.00	79,736	106.05	108.99
III	115.00	2,387.00	81,904	106.18	107.30
IV	114.70	2,560.00	85,285	106.31	110.67
1971					
I	114.40	2,665.00	90,641	106.45	115.84
II	114.12	2,808.00	99,201	106.58	111.81
III	113.09	2,997.00	110,640	106.63	112.74
IV	112.06	3,472.00	125,810	106.63	109.41
1972					
I	111.03	4,121.00	143,192	106.63	99.31
II	110.00	4,850.00	152,942	106.63	93.44
III	110.33	5,439.00	158,827	106.63	94.24
IV	110.66	6,760.00	184,695	106.63	83.51
1973					
I	110.90	8,465.00	217,465	106.63	95.15
II	111.17	9,217.00	243,710	106.63	90.30
III	112.05	11,438.00	270,795	106.63	94.66
IV	112.94	12,001.00	320,470	106.63	90.24
1974					
I	113.82	14,808.00	399,100	106.63	93.11
II	114.71	16,192.00	449,535	106.63	99.61
III	116.18	19,456.00	481,605	106.63	91.77
IV	117.65	24,874.00	545,130	106.63	85.72

Table 7 (Continued

Year	Real GDP[b] Index	CPI[b]	Nominal Money Stock	Population Index	Real Wages[c] Index
1975					
I	119.12	28,574.00	630,675	106.63	86.18
II	120.59	31,240.00	684,825	106.63	89.52
III	121.03	34,963.00	706,045	106.63	80.33
IV	121.47	41,504.00	806,930	106.63	83.02
1976					
I	121.91	42,835.00	984,625	106.63	81.16
II	122.35	45,238.00	1,130,910	106.63	77.03
III	122.70	53,814.00	1,227,526	106.63	78.24
IV	123.00	58,090.00	1,407,090	106.63	76.63